Solid Gold!

This sculpture shows five intersecting tetrahedrons. You can use nets to create models of tetrahedrons and other three-dimensional figures.
Vocabulary
Match each term on the left with a definition on the right.
1. equilateral  A. the distance from the center of a regular polygon to a side of the polygon
2. parallelogram  B. a quadrilateral with four right angles
3. apothem  C. a quadrilateral with two pairs of parallel sides
4. composite figure  D. having all sides congruent
E. a figure made up of simple shapes, such as triangles, rectangles, trapezoids, and circles

Find Area in the Coordinate Plane
Find the area of each figure with the given vertices.
5. \(\triangle ABC\) with \(A(0, 3), B(5, 3),\) and \(C(2, -1)\)
6. rectangle \(KLMN\) with \(K(-2, 3), L(-2, 7), M(6, 7),\) and \(N(6, 3)\)
7. \(\odot P\) with center \(P(2, 3)\) that passes through the point \(Q(-6, 3)\)

Circumference and Area of Circles
Find the circumference and area of each circle. Give your answers in terms of \(\pi\).
8. \(\odot 8 \text{ cm}\)
9. \(\odot 21 \text{ ft}\)
10. \(\odot \frac{32}{\pi} \text{ in.}\)

Distance and Midpoint Formulas
Find the length and midpoint of the segment with the given endpoints.
11. \(A(-3, 2)\) and \(B(5, 6)\)
12. \(C(-4, -4)\) and \(D(2, -3)\)
13. \(E(0, 1)\) and \(F(-3, 4)\)
14. \(G(2, -5)\) and \(H(-2, -2)\)

Evaluate Expressions
Evaluate each expression for the given values of the variables.
15. \(\sqrt{\frac{A}{\pi}}\) for \(A = 121 \pi \text{ cm}^2\)
16. \(\frac{2A}{P}\) for \(A = 128 \text{ ft}^2\) and \(P = 32\) ft
17. \(\sqrt{c^2 - a^2}\) for \(a = 8\) m and \(c = 17\) m
18. \(\frac{2A}{h} - b_1\) for \(A = 60 \text{ in}^2\), \(b_1 = 8\) in., and \(h = 6\) in.
Previously, you
- analyzed properties of figures in a plane.
- found the perimeters and areas of triangles, circles, polygons, and composite figures.
- studied the effects of changing dimensions of polygons proportionally.

You will study
- properties of three-dimensional figures.
- the surface areas and volumes of three-dimensional figures.
- the effects of changing dimensions of three-dimensional figures proportionally.

You can use the skills learned in this chapter
- in all your future math classes, including Precalculus.
- to study other fields such as chemistry, physics, and architecture.
- to solve problems concerning interior design, packaging, and construction.

Key Vocabulary/Vocabulario

<table>
<thead>
<tr>
<th>English</th>
<th>Spanish</th>
</tr>
</thead>
<tbody>
<tr>
<td>cone</td>
<td>cono</td>
</tr>
<tr>
<td>cylinder</td>
<td>cilindro</td>
</tr>
<tr>
<td>net</td>
<td>plantilla</td>
</tr>
<tr>
<td>polyhedron</td>
<td>poliedro</td>
</tr>
<tr>
<td>prism</td>
<td>prisma</td>
</tr>
<tr>
<td>pyramid</td>
<td>pirámide</td>
</tr>
<tr>
<td>sphere</td>
<td>esfera</td>
</tr>
<tr>
<td>surface area</td>
<td>área total</td>
</tr>
<tr>
<td>volume</td>
<td>volumen</td>
</tr>
</tbody>
</table>

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following questions. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word polyhedron begins with the root poly-. List some other words that begin with poly-. What do all of these words have in common?

2. The word cone comes from the root ko-, which means "to sharpen." Think of sharpening a pencil. How do you think this relates to a cone?

3. What does the word surface mean? What do you think the surface area of a three-dimensional figure is?

4. The figure shown is a net for a cube. How do you think a net is related to a three-dimensional object?
Writing Strategy: Draw Three-Dimensional Figures

When you encounter a three-dimensional figure such as a cylinder, cone, sphere, prism, or pyramid, it may help you to make a quick sketch so that you can visualize its shape.

Use these tips to help you draw quick sketches of three-dimensional figures.

Try This

1. Explain and show how to draw a cube, a prism with equal length, width, and height.
2. Draw a prism, starting with two hexagons. (Hint: Draw the hexagons as if you were viewing them at an angle.)
3. Draw a pyramid, starting with a triangle and a point above the triangle.
**Objectives**
Classify three-dimensional figures according to their properties.
Use nets and cross sections to analyze three-dimensional figures.

**Vocabulary**
- face
- edge
- vertex
- prism
- cylinder
- pyramid
- cone
- cube
- net
- cross section

**Why learn this?**
Some farmers in Japan grow cube-shaped watermelons to save space in small refrigerators. Each fruit costs about the equivalent of U.S. $80. (See Example 4.)

Three-dimensional figures, or **solids**, can be made up of flat or curved surfaces. Each flat surface is called a **face**. An **edge** is the segment that is the intersection of two faces. A **vertex** is the point that is the intersection of three or more faces.

**Three-Dimensional Figures**

<table>
<thead>
<tr>
<th>TERM</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A <strong>prism</strong> is formed by two parallel congruent polygonal faces called <strong>bases</strong> connected by faces that are parallelograms.</td>
<td><img src="image" alt="prism" /> Bases</td>
</tr>
<tr>
<td>A <strong>cylinder</strong> is formed by two parallel congruent circular bases and a curved surface that connects the bases.</td>
<td><img src="image" alt="cylinder" /> Bases</td>
</tr>
<tr>
<td>A <strong>pyramid</strong> is formed by a polygonal base and triangular faces that meet at a common vertex.</td>
<td><img src="image" alt="pyramid" /> Vertex Base</td>
</tr>
<tr>
<td>A <strong>cone</strong> is formed by a circular base and a curved surface that connects the base to a vertex.</td>
<td><img src="image" alt="cone" /> Vertex Base</td>
</tr>
</tbody>
</table>

A **cube** is a prism with six square faces. Other prisms and pyramids are named for the shape of their bases.
**Example 1** Classifying Three-Dimensional Figures

Classify each figure. Name the vertices, edges, and bases.

- **A** rectangular pyramid
  - vertices: \( A, B, C, D, E \)
  - edges: \( AB, BC, CD, AD, AE, BE, CE, DE \)
  - base: rectangle \( ABCD \)

- **B** cylinder
  - vertices: none
  - edges: none
  - bases: \( \odot P \) and \( \odot Q \)

**Check It Out!**

Classify each figure. Name the vertices, edges, and bases.

1a.

1b.

A net is a diagram of the surfaces of a three-dimensional figure that can be folded to form the three-dimensional figure. To identify a three-dimensional figure from a net, look at the number of faces and the shape of each face.

**Example 2** Identifying a Three-Dimensional Figure From a Net

Describe the three-dimensional figure that can be made from the given net.

- **A**
  - The net has two congruent triangular faces. The remaining faces are parallelograms, so the net forms a triangular prism.

- **B**
  - The net has one square face. The remaining faces are triangles, so the net forms a square pyramid.

**Check It Out!**

Describe the three-dimensional figure that can be made from the given net.

2a.

2b.
A **cross section** is the intersection of a three-dimensional figure and a plane.

### Example 3
**Describing Cross Sections of Three-Dimensional Figures**

Describe each cross section.

- **A**
  - The cross section is a triangle.

- **B**
  - The cross section is a circle.

### Example 4
**Food Application**

A chef is slicing a cube-shaped watermelon for a buffet. How can the chef cut the watermelon to make a slice of each shape?

- **A** a square
  - Cut parallel to the bases.

- **B** a hexagon
  - Cut through the midpoints of the edges.

### Check It Out

3a. Describe each cross section.

3b. Describe each cross section.

4. How can a chef cut a cube-shaped watermelon to make slices with triangular faces?

### Think and Discuss

1. Compare prisms and cylinders.

2. **Get Organized** Copy and complete the graphic organizer.

---

**Prisms**

- How are they alike?

**Pyramids**

- How are they different?
GUIDED PRACTICE

1. **Vocabulary** A ____?____ has two circular bases. (prism, cylinder, or cone)

Classify each figure. Name the vertices, edges, and bases.

2. 

3. 

4. 

Describe the three-dimensional figure that can be made from the given net.

5. 

6. 

7. 

Describe each cross section.

8. 

9. 

10. 

Art  A sculptor has a cylindrical piece of clay. How can the sculptor slice the clay to make a slice of each given shape?

11. a circle

12. a rectangle

PRACTICE AND PROBLEM SOLVING

Classify each figure. Name the vertices, edges, and bases.

13. 

14. 

15. 

Describe the three-dimensional figure that can be made from the given net.

16. 

17. 

18. 

10-1 Solid Geometry  657
Describe each cross section.

19. 

20. 

21. 

**Architecture** An architect is drawing plans for a building that is a hexagonal prism. How could the architect draw a cutaway of the building that shows a cross section in the shape of each given figure?

22. a hexagon 
23. a rectangle

Name a three-dimensional figure from which a cross section in the given shape can be made.

24. square 
25. rectangle 
26. circle 
27. hexagon

Write a verbal description of each figure.

28. 
29. 
30. 

Draw and label a figure that meets each description.

31. rectangular prism with length 3 cm, width 2 cm, and height 5 cm
32. regular pentagonal prism with side length 6 in. and height 8 in.
33. cylinder with radius 4 m and height 7 m

Draw a net for each three-dimensional figure.

34. 
35. 
36. 

---

**Multi-Step Test Prep**

37. This problem will prepare you for the Multi-Step Test Prep on page 678. A manufacturer of camping gear makes a wall tent in the shape shown in the diagram.

a. Classify the three-dimensional figure that the wall tent forms.

b. What shapes make up the faces of the tent? How many of each shape are there?

c. Draw a net for the wall tent.
38. **ERROR ANALYSIS**  A regular hexagonal prism is intersected by a plane as shown. Which cross section is incorrect? Explain.

39. **Critical Thinking**  A three-dimensional figure has 5 faces. One face is adjacent to every other face. Four of the faces are congruent. Draw a figure that meets these conditions.

40. **Write About It**  Which of the following figures is not a net for a cube? Explain.

41. Which three-dimensional figure does the net represent?

42. Which shape CANNOT be a face of a hexagonal prism?

43. What shape is the cross section formed by a cone and a plane that is perpendicular to the base and that passes through the vertex of the cone?

44. Which shape best represents a hexagonal prism viewed from the top?
**CHALLENGE AND EXTEND**

A *double cone* is formed by two cones that share the same vertex. Sketch each cross section formed by a double cone and a plane.

45.  
46.  
47.  

**Crafts** Elena is designing patterns for gift boxes. Draw a pattern that she can use to create each box. Be sure to include tabs for gluing the sides together.

48. a box that is a square pyramid where each triangular face is an isosceles triangle with a height equal to three times the width

49. a box that is a cylinder with the diameter equal to the height

50. a box that is a rectangular prism with a base that is twice as long as it is wide, and with a rectangular pyramid on the top base

51. A net of a prism is shown. The bases of the prism are regular hexagons, and the rectangular faces are all congruent.
   a. List all pairs of parallel faces in the prism.
   b. Draw a net of a prism with bases that are regular pentagons. How many pairs of parallel faces does the prism have?

**SPIRAL REVIEW**

Write the equation that fits the description. *(Previous course)*

52. the equation of the graph that is the reflection of the graph of \( y = x^2 \) over the \( x\)-axis

53. the equation of the graph of \( y = x^2 \) after a vertical translation of 6 units upward

54. the quadratic equation of a graph that opens upward and is wider than \( y = x^2 \)

Name the largest and smallest angles of each triangle. *(Lesson 5-5)*

55.  
56.  
57.  

Determine whether the two polygons are similar. If so, give the similarity ratio. *(Lesson 7-2)*

58.  
59.
10-2 Representations of Three-Dimensional Figures

**Objectives**
- Draw representations of three-dimensional figures.
- Recognize a three-dimensional figure from a given representation.

**Vocabulary**
- orthographic drawing
- isometric drawing
- perspective drawing
- vanishing point
- horizon

**Who uses this?**
Architects make many different kinds of drawings to represent three-dimensional figures. (See Exercise 34.)

There are many ways to represent a three-dimensional object. An **orthographic drawing** shows six different views of an object: top, bottom, front, back, left side, and right side.

**Example 1**

**Drawing Orthographic Views of an Object**

Draw all six orthographic views of the given object. Assume there are no hidden cubes.

- **Top:**
- **Bottom:**
- **Front:**
- **Back:**
- **Left:**
- **Right:**

1. Draw all six orthographic views of the given object. Assume there are no hidden cubes.
**Isometric drawing** is a way to show three sides of a figure from a corner view. You can use *isometric dot paper* to make an isometric drawing. This paper has diagonal rows of dots that are equally spaced in a repeating triangular pattern.

**Example 2**

**Drawing an Isometric View of an Object**

Draw an isometric view of the given object. Assume there are no hidden cubes.

---

2. Draw an isometric view of the given object. Assume there are no hidden cubes.

In a **perspective drawing**, nonvertical parallel lines are drawn so that they meet at a point called a **vanishing point**. Vanishing points are located on a horizontal line called the **horizon**. A one-point perspective drawing contains one vanishing point. A two-point perspective drawing contains two vanishing points.

---

**Student to Student**

**Perspective Drawing**

*When making a perspective drawing, it helps me to remember that all vertical lines on the object will be vertical in the drawing.*

---

*Jacob Martin*
MacArthur High School
### Drawing an Object in Perspective

#### A
Draw a cube in one-point perspective.

1. Draw a horizontal line to represent the horizon. Mark a vanishing point on the horizon. Then draw a square below the horizon. This is the front of the cube.
2. From each corner of the square, lightly draw dashed segments to the vanishing point.
3. Lightly draw a smaller square with vertices on the dashed segments. This is the back of the cube.
4. Draw the edges of the cube, using dashed segments for hidden edges. Erase any segments that are not part of the cube.

#### B
Draw a rectangular prism in two-point perspective.

1. Draw a horizontal line to represent the horizon. Mark two vanishing points on the horizon. Then draw a vertical segment below the horizon and between the vanishing points. This is the front edge of the prism.
2. From each endpoint of the segment, lightly draw dashed segments to each vanishing point.
3. Draw two vertical segments connecting the dashed lines. These are other vertical edges of the prism.
4. Lightly draw dashed segments from each endpoint of the two vertical segments to the vanishing points.
5. Draw the edges of the prism, using dashed lines for hidden edges. Erase any lines that are not part of the prism.

**Helpful Hint**
In a one-point perspective drawing of a cube, you are looking at a face. In a two-point perspective drawing, you are looking at a corner.

### Check It Out!

3a. Draw the block letter L in one-point perspective.
3b. Draw the block letter L in two-point perspective.
**Example 4**

Relating Different Representations of an Object

Determine whether each drawing represents the given object. Assume there are no hidden cubes.

**A**

Yes; the drawing is a one-point perspective view of the object.

**B**

No; the figure in the drawing is made up of four cubes, and the object is made up of only three cubes.

**C**

No; the cubes that share a face in the object do not share a face in the drawing.

**D**

Yes; the drawing shows the six orthographic views of the object.

4. Determine whether the drawing represents the given object. Assume there are no hidden cubes.

**Think and Discuss**

1. Describe the six orthographic views of a cube.

2. In a perspective drawing, are all parallel lines drawn so that they meet at a vanishing point? Why or why not?

3. GET ORGANIZED Copy and complete the graphic organizer.

<table>
<thead>
<tr>
<th>Type of Drawing</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top, bottom, front, back, left, and right views</td>
<td>Parallel lines are drawn so that they meet at vanishing point(s).</td>
</tr>
</tbody>
</table>
GUIDED PRACTICE

1. **Vocabulary**  In a(n) ? drawing, the *vanishing points* are located on the *horizon*. (orthographic, isometric, or perspective)

   Draw all six orthographic views of each object. Assume there are no hidden cubes.

   2. 
   3. 
   4. 

SEE EXAMPLE 1  

p. 661

SEE EXAMPLE 2  

Draw an isometric view of each object. 
Assume there are no hidden cubes.

5. 
6. 
7. 

SEE EXAMPLE 3  

p. 663

Draw each object in one-point and two-point perspectives. 
Assume there are no hidden cubes.

8. rectangular prism  
9. block letter □

SEE EXAMPLE 4  

p. 664

Determine whether each drawing represents the given object. Assume there are no hidden cubes.

10. 
11. 

12. 

13. 

PRACTICE AND PROBLEM SOLVING

Draw all six orthographic views of each object. Assume there are no hidden cubes.

14. 
15. 
16.
Draw an isometric view of each object. Assume there are no hidden cubes.

17. 

18. 

19. 

Draw each object in one-point and two-point perspective. Assume there are no hidden cubes.

20. right triangular prism

21. block letter \( \Box \)

Determine whether each drawing represents the given object. Assume there are no hidden cubes.

22. 

23. 

24. 

25. 

26. Use the top, front, side, and isometric views to build the three-dimensional figure out of unit cubes. Then draw the figure in one-point perspective.

27. 

28. 

Use the top, side, and front views to draw an isometric view of each figure.

29. This problem will prepare you for the Multi-Step Test Prep on page 678. A camping gear catalog shows the three given views of a tent.

a. Draw a bottom view of the tent.

b. Make a sketch of the tent.

c. Each edge of the three-dimensional figure from part b represents one pole of the tent. How many poles does this tent have?
Draw all six orthographic views of each object.

30. 

31. 

32. 

33. **Critical Thinking** Describe or draw two figures that have the same left, right, front, and back orthographic views but have different top and bottom views.

34. **Architecture** Perspective drawings are used by architects to show what a finished room will look like.
   
   **a.** Is the architect’s sketch in one-point or two-point perspective?
   
   **b. Write About It** How would you locate the vanishing point(s) in the architect’s sketch?

35. Which three-dimensional figure has these three views?

A. 

B. 

C. 

D. 

36. Which drawing best represents the top view of the three-dimensional figure?

E. 

F. 

G. 

H. 

37. **Short Response** Draw a one-point perspective view and an isometric view of a triangular prism. Explain how the two drawings are different.
CHALLENGE AND EXTEND

Draw each figure using one-point perspective. *(Hint: First lightly draw a rectangular prism. Enclose the figure in the prism.)*

38. an octagonal prism  
39. a cylinder  
40. a cone

41. A frustum of a cone is a part of a cone with two parallel bases. Copy the diagram of the frustum of a cone.
   a. Draw the entire cone.
   b. Draw all six orthographic views of the frustum.
   c. Draw a net for the frustum.

42. Art  Draw a one-point or two-point perspective drawing of the inside of a room. Include at least two pieces of furniture drawn in perspective.

SPIRAL REVIEW

Find the two numbers. *(Previous course)*

43. The sum of two numbers is 30. The difference between 2 times the first number and 2 times the second number is 20.

44. The difference between the first number and the second number is 7. When the second number is added to 4 times the first number, the result is 38.

45. The second number is 5 more than the first number. Their sum is 5.

For \(A(4, 2), B(6, 1), C(3, 0),\) and \(D(2, 0),\) find the slope of each line. *(Lesson 3-5)*

46. \(\overleftrightarrow{AB}\)  
47. \(\overleftrightarrow{AC}\)  
48. \(\overleftrightarrow{AD}\)

Describe the faces of each figure. *(Lesson 10-1)*

49. pentagonal prism  
50. cube  
51. triangular pyramid

Using Technology

You can use geometry software to draw figures in one- and two-point perspectives.

1. a. Draw a horizontal line to represent the horizon. Create a vanishing point on the horizon. Draw a rectangle with two sides parallel to the horizon. Draw a segment from each vertex to the vanishing point.

   b. Draw a smaller rectangle with vertices on the segments that intersect the horizon. Hide these segments and complete the figure.

   c. Drag the vanishing point to different locations on the horizon. Describe what happens to the figure.

2. Describe how you would use geometry software to draw a figure in two-point perspective.
10-3

Use Nets to Create Polyhedrons

A polyhedron is formed by four or more polygons that intersect only at their edges. The faces of a regular polyhedron are all congruent regular polygons, and the same number of faces intersect at each vertex. Regular polyhedra are also called Platonic solids. There are exactly five regular polyhedra.

**Activity**

Use geometry software or a compass and straightedge to create a larger version of each net on heavy paper. Fold each net into a polyhedron.

<table>
<thead>
<tr>
<th>REGULAR POLYHEDRONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAME</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>Tetrahedron</td>
</tr>
<tr>
<td>Octahedron</td>
</tr>
<tr>
<td>Icosahedron</td>
</tr>
<tr>
<td>Cube</td>
</tr>
<tr>
<td>Dodecahedron</td>
</tr>
</tbody>
</table>

**Try This**

1. Complete the table for the number of vertices $V$, edges $E$, and faces $F$ for each of the polyhedrons you made in Activity 1.

2. Make a Conjecture What do you think is true about the relationship between the number of vertices, edges, and faces of a polyhedron?
**Objectives**

Apply Euler’s formula to find the number of vertices, edges, and faces of a polyhedron.

Develop and apply the distance and midpoint formulas in three dimensions.

**Vocabulary**

polyhedron

space

---

**Why learn this?**

Divers can use a three-dimensional coordinate system to find distances between two points under water. (See Example 5.)

A **polyhedron** is formed by four or more polygons that intersect only at their edges. Prisms and pyramids are polyhedrons, but cylinders and cones are not.

<table>
<thead>
<tr>
<th>Polyhedrons</th>
<th>Not polyhedrons</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="Image" alt="Polyhedron Images" /></td>
<td><img src="Image" alt="Not Polyhedron Images" /></td>
</tr>
</tbody>
</table>

In the lab before this lesson, you made a conjecture about the relationship between the vertices, edges, and faces of a polyhedron. One way to state this relationship is given below.

**Euler’s Formula**

For any polyhedron with $V$ vertices, $E$ edges, and $F$ faces, $V - E + F = 2$.

---

**Example 1**

**Using Euler’s Formula**

Find the number of vertices, edges, and faces of each polyhedron. Use your results to verify Euler’s formula.

**A**

$V = 4$, $E = 6$, $F = 4$

$4 - 6 + 4 = 2$

$2 = 2$

**B**

$V = 10$, $E = 15$, $F = 7$

$10 - 15 + 7 = 2$

$2 = 2$

---

**Check It Out!**

Find the number of vertices, edges, and faces of the polyhedron. Use your results to verify Euler’s formula.

1a.

1b.
A diagonal of a three-dimensional figure connects two vertices of two different faces. Diagonal \(d\) of a rectangular prism is shown in the diagram. By the Pythagorean Theorem, \(\ell^2 + w^2 = x^2\), and \(x^2 + h^2 = d^2\). Using substitution, \(\ell^2 + w^2 + h^2 = d^2\).

**Diagonal of a Right Rectangular Prism**

The length of a diagonal \(d\) of a right rectangular prism with length \(\ell\), width \(w\), and height \(h\) is \(d = \sqrt{\ell^2 + w^2 + h^2}\).

**Example 2**

Using the Pythagorean Theorem in Three Dimensions

Find the unknown dimension in each figure.

**A**  
The length of the diagonal of a 3 in. by 4 in. by 5 in. rectangular prism

\[d = \sqrt{3^2 + 4^2 + 5^2} \quad \text{Substitute 3 for } \ell, 4 \text{ for } w, \text{ and } 5 \text{ for } h.\]

\[= \sqrt{9 + 16 + 25} \quad \text{Simplify.}\]

\[= \sqrt{50} \approx 7.1 \text{ in.}\]

**B**  
The height of a rectangular prism with an 8 ft by 12 ft base and an 18 ft diagonal

\[18 = \sqrt{8^2 + 12^2 + h^2} \quad \text{Substitute 18 for } d, 8 \text{ for } \ell, \text{ and } 12 \text{ for } w.\]

\[18^2 = (\sqrt{8^2 + 12^2 + h^2})^2 \quad \text{Square both sides of the equation.}\]

\[324 = 64 + 144 + h^2 \quad \text{Simplify.}\]

\[h^2 = 116 \quad \text{Solve for } h^2.\]

\[h = \sqrt{116} \approx 10.8 \text{ ft} \quad \text{Take the square root of both sides.}\]

**Check it out!**

2. Find the length of the diagonal of a cube with edge length 5 cm.

**Space**

is the set of all points in three dimensions. Three coordinates are needed to locate a point in space. A three-dimensional coordinate system has 3 perpendicular axes: the \(x\)-axis, the \(y\)-axis, and the \(z\)-axis. An ordered triple \((x, y, z)\) is used to locate a point. To locate the point \((3, 2, 4)\), start at \((0, 0, 0)\). From there move 3 units forward, 2 units right, and then 4 units up.

**Example 3**

Graphing Figures in Three Dimensions

Graph each figure.

**A**  
A cube with edge length 4 units and one vertex at \((0, 0, 0)\)

The cube has 8 vertices:

\((0, 0, 0), (0, 4, 0), (0, 0, 4), (4, 0, 0), (4, 4, 0), (4, 0, 4), (0, 4, 4), (4, 4, 4)\).
Graph each figure.

**B** a cylinder with radius 3 units, height 5 units, and one base centered at $(0, 0, 0)$

Graph the center of the bottom base at $(0, 0, 0)$.

Since the height is 5, graph the center of the top base at $(0, 0, 5)$.

The radius is 3, so the bottom base will cross the $x$-axis at $(3, 0, 0)$ and the $y$-axis at $(0, 3, 0)$.

Draw the top base parallel to the bottom base and connect the bases.

3. Graph a cone with radius 5 units, height 7 units, and the base centered at $(0, 0, 0)$.

You can find the distance between the two points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ by drawing a rectangular prism with the given points as endpoints of a diagonal. Then use the formula for the length of the diagonal. You can also use a formula related to the Distance Formula. (See Lesson 1-6.)

The formula for the midpoint between $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is related to the Midpoint Formula. (See Lesson 1-6.)

### Distance and Midpoint Formulas in Three Dimensions

The distance between the points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$ 

The midpoint of the segment with endpoints $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

**Example 4** Finding Distances and Midpoints in Three Dimensions

Find the distance between the given points. Find the midpoint of the segment with the given endpoints. Round to the nearest tenth, if necessary.

A $(0, 0, 0)$ and $(3, 4, 12)$

distance:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{(3 - 0)^2 + (4 - 0)^2 + (12 - 0)^2}$$

$$= \sqrt{9 + 16 + 144}$$

$$= \sqrt{169} = 13 \text{ units}$$
Find the distance between the given points. Find the midpoint of the segment with the given endpoints. Round to the nearest tenth, if necessary.

### Example 5

**Recreation Application**

Two divers swam from a boat to the locations shown in the diagram. How far apart are the divers?

The location of the boat can be represented by the ordered triple \((0, 0, 0)\), and the locations of the divers can be represented by the ordered triples \((18, 9, -8)\) and \((-15, -6, -12)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}
\]
\[
= \sqrt{(-15 - 18)^2 + (-6 - 9)^2 + (-12 + 8)^2}
\]
\[
= \sqrt{1330} \approx 36.5 \text{ ft}
\]

### Check It Out!

5. **What if...?** If both divers swam straight up to the surface, how far apart would they be?

---

**THINK AND DISCUSS**

1. Explain how to find the distance between two points in a three-dimensional coordinate system.
2. **GET ORGANIZED** Copy and complete the graphic organizer.
10-3 Exercises

**GUIDED PRACTICE**

1. **Vocabulary** Explain why a cylinder is not a **polyhedron**.

Find the number of vertices, edges, and faces of each polyhedron. Use your results to verify Euler's formula.

2. 

3. 

4. 

Find the unknown dimension in each figure. Round to the nearest tenth, if necessary.

5. the length of the diagonal of a 4 ft by 8 ft by 12 ft rectangular prism

6. the height of a rectangular prism with a 6 in. by 10 in. base and a 13 in. diagonal

7. the length of the diagonal of a square prism with a base edge length of 12 in. and a height of 1 in.

Graph each figure.

8. a cone with radius 8 units, height 4 units, and the base centered at (0, 0, 0)

9. a cylinder with radius 3 units, height 4 units, and one base centered at (0, 0, 0)

10. a cube with edge length 7 units and one vertex at (0, 0, 0)

Find the distance between the given points. Find the midpoint of the segment with the given endpoints. Round to the nearest tenth, if necessary.

11. (0, 0, 0) and (5, 9, 10)  
12. (0, 3, 8) and (7, 0, 14)  
13. (4, 6, 10) and (9, 12, 15)

14. **Recreation** After a day hike, a group of hikers set up a camp 3 km east and 7 km north of the starting point. The elevation of the camp is 0.6 km higher than the starting point. What is the distance from the camp to the starting point?

**PRACTICE AND PROBLEM SOLVING**

Find the number of vertices, edges, and faces of each polyhedron. Use your results to verify Euler's formula.

15. 

16. 

17. 

Find the unknown dimension in each figure. Round to the nearest tenth, if necessary.

18. the length of the diagonal of a 7 yd by 8 yd by 16 yd rectangular prism

19. the height of a rectangular prism with a 15 m by 6 m base and a 17 m diagonal

20. the edge length of a cube with an 8 cm diagonal
Graph each figure.

21. a cylinder with radius 5 units, height 3 units, and one base centered at (0, 0, 0)

22. a cone with radius 2 units, height 4 units, and the base centered at (0, 0, 0)

23. a square prism with base edge length 5 units, height 3 units, and one vertex at (0, 0, 0)

Find the distance between the given points. Find the midpoint of the segment with the given endpoints. Round to the nearest tenth, if necessary.

24. (0, 0, 0) and (4, 4, 4)  
25. (2, 3, 7) and (9, 10, 10)  
26. (2, 5, 3) and (8, 8, 10)

27. **Meteorology** A cloud has an elevation of 6500 feet. A raindrop falling from the cloud was blown 700 feet south and 500 feet east before it hit the ground. How far did the raindrop travel from the cloud to the ground?

28. **Multi-Step** Find the length of a diagonal of the rectangular prism at right. If the length, width, and height are doubled, what happens to the length of the diagonal?

For each three-dimensional figure, find the missing value and draw a figure with the correct number of vertices, edges, and faces.

<table>
<thead>
<tr>
<th>Vertices V</th>
<th>Edges E</th>
<th>Faces F</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>8</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>29.</td>
<td>8</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>30.</td>
<td>9</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>31.</td>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

32. **Algebra** Each base of a prism is a polygon with \( n \) sides. Write an expression for the number of vertices \( V \), the number of edges \( E \), and the number of faces \( F \) in terms of \( n \). Use your results to show that Euler’s formula is true for all prisms.

33. **Algebra** The base of a pyramid is a polygon with \( n \) sides. Write an expression for the number of vertices \( V \), the number of edges \( E \), and the number of faces \( F \) in terms of \( n \). Use your results to show that Euler’s formula is true for all pyramids.

34. **Multi-Step Test Prep** on page 678.  
The tent at right is a triangular prism where \( \overline{NM} \cong \overline{NP} \) and \( \overline{KJ} \cong \overline{KL} \) and has the given dimensions.

a. The tent manufacturer sets up the tent on a coordinate system so that \( J \) is at the origin and \( M \) has coordinates \((7, 0, 0)\). Find the coordinates of the other vertices.

b. The manufacturer wants to know the distance from \( K \) to \( P \) in order to make an extra support pole for the tent. Find \( KP \) to the nearest tenth.
Find the missing dimension of each rectangular prism. Give your answers in simplest radical form.

<table>
<thead>
<tr>
<th>Length $\ell$</th>
<th>Width $w$</th>
<th>Height $h$</th>
<th>Diagonal $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35. 6 in.</td>
<td>6 in.</td>
<td>6 in.</td>
<td></td>
</tr>
<tr>
<td>36. 24</td>
<td></td>
<td>60</td>
<td>65</td>
</tr>
<tr>
<td>37. 12</td>
<td>18</td>
<td></td>
<td>24</td>
</tr>
<tr>
<td>38.</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Graph each figure.

39. a cylinder with radius 4 units, height 5 units, and one base centered at $(1, 2, 5)$
40. a cone with radius 3 units, height 7 units, and the base centered at $(3, 2, 6)$
41. a cube with edge length 6 units and one vertex at $(4, 2, 3)$
42. a rectangular prism with vertices at $(4, 2, 5)$, $(4, 6, 5)$, $(4, 6, 8)$, $(8, 6, 5)$, $(8, 2, 5)$, $(8, 6, 8)$, $(4, 2, 8)$, and $(8, 2, 8)$
43. a cone with radius 4 units, the vertex at $(4, 7, 8)$, and the base centered at $(4, 7, 1)$
44. a cylinder with a radius of 5 units and bases centered at $(2, 3, 7)$ and $(2, 3, 15)$

Graph each segment with the given endpoints in a three-dimensional coordinate system. Find the length and midpoint of each segment.

45. $(1, 2, 3)$ and $(3, 2, 1)$
46. $(4, 3, 3)$ and $(7, 4, 4)$
47. $(4, 7, 8)$ and $(3, 1, 5)$
48. $(0, 0, 0)$ and $(8, 3, 6)$
49. $(6, 1, 8)$ and $(2, 2, 6)$
50. $(2, 8, 5)$ and $(3, 6, 3)$
51. **Multi-Step** Find $z$ if the distance between $R(6, -1, -3)$ and $S(3, 3, 2)$ is 13.
52. Draw a figure with 6 vertices and 6 faces.

53. **Estimation** Measure the net for a rectangular prism and estimate the length of a diagonal.

54. **Make a Conjecture** What do you think is the longest segment joining two points on a rectangular prism? Test your conjecture using at least three segments whose endpoints are on the prism with vertices $A(0, 0, 0)$, $B(1, 0, 0)$, $C(2, 0, 0)$, $D(0, 2, 0)$, $E(0, 0, 2)$, $F(1, 0, 2)$, $G(1, 2, 2)$, and $H(0, 2, 2)$.

55. **Critical Thinking** The points $A(3, 2, -3)$, $B(5, 8, 6)$, and $C(-3, -5, 3)$ form a triangle. Classify the triangle by sides and angles.

56. **Write About It** A cylinder has a radius of 4 cm and a height of 6 cm. What is the length of the longest segment with both endpoints on the cylinder? Describe the location of the endpoints and explain why it is the longest possible segment.
57. How many faces, edges, and vertices does a hexagonal pyramid have?
   - A) 6 faces, 10 edges, 6 vertices
   - B) 7 faces, 10 edges, 7 vertices
   - C) 7 faces, 12 edges, 7 vertices
   - D) 8 faces, 18 edges, 12 vertices

58. Which is closest to the length of the diagonal of the rectangular prism with length 12 m, width 8 m, and height 6 m?
   - F) 6.6 m
   - G) 44 m
   - H) 15.6 m
   - I) 244.0 m

59. What is the distance between the points \((7, 14, 8)\) and \((9, 3, 12)\) to the nearest tenth?
   - A) 10.9
   - B) 11.9
   - C) 119.0
   - D) 141.0

**CHALLENGE AND EXTEND**

60. **Multi-Step** The bases of the right hexagonal prism are regular hexagons with side length \(a\), and the height of the prism is \(h\). Find the length of the indicated diagonal in terms of \(a\) and \(h\).

61. Determine if the points \(A(-1, 2, 4), B(1, -2, 6)\), and \(C(3, -6, 8)\) are collinear.

62. **Algebra** Write a coordinate proof of the Midpoint Formula using the Distance Formula.

   **Given:** points \(A(x_1, y_1, z_1), B(x_2, y_2, z_2)\), and \(M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)\)

   **Prove:** \(A, B,\) and \(M\) are collinear, and \(AM = MB\).

63. **Algebra** Write a coordinate proof that the diagonals of a rectangular prism are congruent and bisect each other.

   **Given:** a rectangular prism with vertices \(A(0, 0, 0), B(a, 0, 0), C(a, b, 0), D(0, b, 0), E(0, 0, c), F(a, 0, c), G(a, b, c),\) and \(H(0, b, c)\)

   **Prove:** \(AG\) and \(BH\) are congruent and bisect each other.

**SPIRAL REVIEW**

The histogram shows the number of people by age group who attended a natural history museum opening. Find the following.

*(Previous course)*

64. the number of people between 10 and 29 years of age that were in attendance

65. the age group that had the greatest number of people in attendance

Write a formula for the area of each figure after the given change. *(Lesson 9-5)*

66. A parallelogram with base \(b\) and height \(h\) has its height doubled.

67. A trapezoid with height \(h\) and bases \(b_1\) and \(b_2\) has its base \(b_1\) multiplied by \(\frac{1}{2}\).

68. A circle with radius \(r\) has its radius tripled.

Use the diagram for Exercises 69–71. *(Lesson 10-1)*

69. Classify the figure. 70. Name the edges. 71. Name the base.
Three-Dimensional Figures

Your Two Tents  A manufacturer of camping gear offers two types of tents: an A-frame tent and a pyramid tent.

1. The manufacturer’s catalog shows the top, front, and side views of each tent. It shows a two-dimensional shape for each that can be folded to form the three-dimensional shape of the tent. Draw the catalog display for each tent.

The manufacturer uses a three-dimensional coordinate system to represent the vertices of each tent. Each unit of the coordinate system represents one foot.

2. Which tent offers a greater sleeping area?

3. Compare the heights of the tents. Which tent offers more headroom?

4. A camper wants to purchase the tent that has shorter support poles so that she can fit the folded tent in her car more easily. Find the length of pole $\overline{EF}$ in the A-frame tent and the length of pole $\overline{TR}$ in the pyramid tent. Which tent should the camper buy?

<table>
<thead>
<tr>
<th>A-Frame Tent</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$(0, 0, 0)$</td>
</tr>
<tr>
<td>$B$</td>
<td>$(0, 7, 0)$</td>
</tr>
<tr>
<td>$C$</td>
<td>$(0, 3.5, 7)$</td>
</tr>
<tr>
<td>$D$</td>
<td>$(8, 0, 0)$</td>
</tr>
<tr>
<td>$E$</td>
<td>$(8, 7, 0)$</td>
</tr>
<tr>
<td>$F$</td>
<td>$(8, 3.5, 7)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pyramid Tent</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>$(0, 0, 0)$</td>
</tr>
<tr>
<td>$Q$</td>
<td>$(8, 0, 0)$</td>
</tr>
<tr>
<td>$R$</td>
<td>$(8, 8, 0)$</td>
</tr>
<tr>
<td>$S$</td>
<td>$(0, 8, 0)$</td>
</tr>
<tr>
<td>$T$</td>
<td>$(4, 4, 8)$</td>
</tr>
</tbody>
</table>
Quiz for Lessons 10-1 Through 10-3

10-1 Solid Geometry
Classify each figure. Name the vertices, edges, and bases.

1. [Figure 1]
2. [Figure 2]
3. [Figure 3]

Describe the three-dimensional figure that can be made from the given net.

4. [Net 1]
5. [Net 2]
6. [Net 3]

Describe each cross section.

7. [Cross Section 1]
8. [Cross Section 2]
9. [Cross Section 3]

10-2 Representations of Three-Dimensional Figures
Use the figure made of unit cubes for Problems 10 and 11. Assume there are no hidden cubes.

10. Draw all six orthographic views.
11. Draw an isometric view.
12. Draw the block letter T in one-point perspective.
13. Draw the block letter T in two-point perspective.

10-3 Formulas in Three Dimensions
Find the number of vertices, edges, and faces of each polyhedron. Use your results to verify Euler's formula.

14. a square prism
15. a hexagonal pyramid
16. a triangular pyramid
17. A bird flies from its nest to a point that is 6 feet north, 7 feet west, and 6 feet higher in the tree than the nest. How far is the bird from the nest?

Find the distance between the given points. Find the midpoint of the segment with the given endpoints. Round to the nearest tenth, if necessary.

18. (0, 0, 0) and (4, 6, 12)
19. (3, 1, −2) and (5, −5, 7)
20. (3, 5, 9) and (7, 2, 0)
Objectives
Learn and apply the formula for the surface area of a prism.
Learn and apply the formula for the surface area of a cylinder.

Vocabulary
lateral face
lateral edge
right prism
oblique prism
altitude
surface area
lateral surface
axis of a cylinder
right cylinder
oblique cylinder

Why learn this?
The surface area of ice affects how fast it will melt. If the surface exposed to the air is increased, the ice will melt faster. (See Example 5.)

Prisms and cylinders have 2 congruent parallel bases. A **lateral face** is not a base. The edges of the base are called **base edges**. A **lateral edge** is not an edge of a base.

The lateral faces of a **right prism** are all rectangles. An **oblique prism** has at least one nonrectangular lateral face.

An **altitude** of a prism or cylinder is a perpendicular segment joining the planes of the bases. The **height** of a three-dimensional figure is the length of an altitude.

**Surface area** is the total area of all faces and curved surfaces of a three-dimensional figure. The **lateral area** of a prism is the sum of the areas of the lateral faces.

The net of a right prism can be drawn so that the lateral faces form a rectangle with the same height as the prism. The base of the rectangle is equal to the perimeter of the base of the prism.

**Lateral Area and Surface Area of Right Prisms**

The lateral area of a right prism with base perimeter $P$ and height $h$ is $L = Ph$.

The surface area of a right prism with lateral area $L$ and base area $B$ is $S = L + 2B$, or $S = Ph + 2B$.

The surface area of a cube with edge length $s$ is $S = 6s^2$.

The surface area of a right rectangular prism with length $l$, width $w$, and height $h$ can be written as $S = 2lw + 2wh + 2lh$. 
### Example 1

**Finding Lateral Areas and Surface Areas of Prisms**

Find the lateral area and surface area of each right prism. Round to the nearest tenth, if necessary.

A **the rectangular prism**

\[
L = Ph \\
= (28)(12) = 336 \text{ cm}^2 \\
S = Ph + 2B \\
= 336 + 2(6)(8) \\
= 432 \text{ cm}^2
\]

B **the regular hexagonal prism**

\[
L = Ph \\
= 36(10) = 360 \text{ m}^2 \\
S = Ph + 2B \\
= 360 + 2\left(54\sqrt{3}\right) \\
= 547.1 \text{ m}^2
\]

The base area is \(B = \frac{1}{2}aP = 54\sqrt{3} \text{ m}\).

### Check It Out!

1. Find the lateral area and surface area of a cube with edge length 8 cm.

The **lateral surface** of a cylinder is the curved surface that connects the two bases. The **axis of a cylinder** is the segment with endpoints at the centers of the bases. The axis of a **right cylinder** is perpendicular to its bases. The axis of an **oblique cylinder** is not perpendicular to its bases. The altitude of a right cylinder is the same length as the axis.

### Lateral Area and Surface Area of Right Cylinders

The lateral area of a right cylinder with radius \(r\) and height \(h\) is \(L = 2\pi rh\).

The surface area of a right cylinder with lateral area \(L\) and base area \(B\) is \(S = L + 2B\), or \(S = 2\pi rh + 2\pi r^2\).
**Example 2** Finding Lateral Areas and Surface Areas of Right Cylinders

Find the lateral area and surface area of each right cylinder. Give your answers in terms of \( \pi \).

**A**

![Diagram of a right cylinder with dimensions 2 m and 5 m]

\[
L = 2\pi rh = 2\pi (1)(5) = 10\pi \text{ m}^2 \quad \text{The radius is half the diameter, or 1 m.}
\]

\[
S = L + 2\pi r^2 = 10\pi + 2\pi (1)^2 = 12\pi \text{ m}^2
\]

**B**

A cylinder with a circumference of \( 10\pi \) cm and a height equal to 3 times the radius.

**Step 1** Use the circumference to find the radius.

\[
C = 2\pi r \quad \text{Circumference of a circle}
\]

\[
10\pi = 2\pi r \quad \text{Substitute 10\pi for C.}
\]

\[
r = 5 \quad \text{Divide both sides by 2\pi.}
\]

**Step 2** Use the radius to find the lateral area and surface area.

The height is 3 times the radius, or 15 cm.

\[
L = 2\pi rh = 2\pi (5)(15) = 150\pi \text{ cm}^2 \quad \text{Lateral area}
\]

\[
S = 2\pi rh + 2\pi r^2 = 150\pi + 2\pi (5)^2 = 200\pi \text{ cm}^2 \quad \text{Surface area}
\]

**Example 3** Finding Surface Areas of Composite Three-Dimensional Figures

Find the surface area of the composite figure. Round to the nearest tenth.

Remember!

Always round at the last step of the problem. Use the value of \( \pi \) given by the \( \pi \) key on your calculator.

The surface area of the right rectangular prism is

\[
S = Ph + 2B = 80(20) + 2(24)(16) = 2368 \text{ ft}^2.
\]

A right cylinder is removed from the rectangular prism. The lateral area is \( L = 2\pi rh = 2\pi (4)(20) = 160\pi \text{ ft}^2 \). The area of each base is \( B = \pi r^2 = \pi (4)^2 = 16\pi \text{ ft}^2 \).

The surface area of the composite figure is the sum of the areas of all surfaces on the exterior of the figure.

\[
S = (\text{prism surface area}) + (\text{cylinder lateral area}) - (\text{cylinder base area})
\]

\[
= 2368 + 160\pi - 2(16\pi)
\]

\[
= 2368 + 128\pi \approx 2770.1 \text{ ft}^2
\]

**Check It Out!**

3. Find the surface area of the composite figure. Round to the nearest tenth.
**Example 4**

Exploring Effects of Changing Dimensions

The length, width, and height of the right rectangular prism are doubled. Describe the effect on the surface area.

<table>
<thead>
<tr>
<th>Original dimensions:</th>
<th>Length, width, and height doubled:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = Ph + 2B$</td>
<td>$S = Ph + 2B$</td>
</tr>
<tr>
<td>$= 16(3) + 2(6)(2)$</td>
<td>$= 32(6) + 2(12)(4)$</td>
</tr>
<tr>
<td>$= 72$ in$^2$</td>
<td>$= 288$ in$^2$</td>
</tr>
</tbody>
</table>

Notice that $288 = 4(72)$. If the length, width, and height are doubled, the surface area is multiplied by $2^2$, or 4.

4. The height and diameter of the cylinder are multiplied by $\frac{1}{2}$. Describe the effect on the surface area.

**Example 5**

Chemistry Application

If two pieces of ice have the same volume, the one with the greater surface area will melt faster because more of it is exposed to the air. One piece of ice shown is a rectangular prism, and the other is half a cylinder. Given that the volumes are approximately equal, which will melt faster?

rectangular prism:

$$S = Ph + 2B = 12(3) + 2(8) = 52 \text{ cm}^2$$

half cylinder:

$$S = \pi rh + \pi r^2 + 2rh = \pi(4)(1) + \pi(4)^2 + 8(1)$$
$$= 20\pi + 8 \approx 70.8 \text{ cm}^2$$

The half cylinder of ice will melt faster.

Use the information above to answer the following.

5. A piece of ice shaped like a 5 cm by 5 cm by 1 cm rectangular prism has approximately the same volume as the pieces above. Compare the surface areas. Which will melt faster?

**Think and Discuss**

1. Explain how to find the surface area of a cylinder if you know the lateral area and the radius of the base.

2. Describe the difference between an oblique prism and a right prism.

3. **Get Organized** Copy and complete the graphic organizer. Write the formulas in each box.
10-4 Exercises

**GUIDED PRACTICE**

1. **Vocabulary** How many lateral faces does a pentagonal prism have?

Find the lateral area and surface area of each right prism.

2. [Diagram of a rectangular prism]

3. [Diagram of a triangular prism]

4. a cube with edge length 9 inches

Find the lateral area and surface area of each right cylinder. Give your answers in terms of \( \pi \).

5. [Diagram of a cylinder with dimensions]

6. [Diagram of a cylinder with dimensions]

7. a cylinder with base area \( 64\pi \text{ m}^2 \) and a height 3 meters less than the radius

**Multi-Step** Find the surface area of each composite figure. Round to the nearest tenth.

8. [Diagram of a composite figure]

9. [Diagram of a composite figure]

Describe the effect of each change on the surface area of the given figure.

10. The dimensions are cut in half.

11. The dimensions are multiplied by \( \frac{2}{3} \).

12. **Consumer Application** The greater the lateral area of a florescent light bulb, the more light the bulb produces. One cylindrical light bulb is 16 inches long with a 1-inch radius. Another cylindrical light bulb is 23 inches long with a \( \frac{3}{4} \)-inch radius. Which bulb will produce more light?
PRACTICE AND PROBLEM SOLVING

Independent Practice

Find the lateral area and surface area of each right prism. Round to the nearest tenth, if necessary.

13. \( \text{Lateral Area} \): 330 \( \text{cm}^2 \) \( \text{Surface Area} \): 430 \( \text{cm}^2 \)

14. \( \text{Lateral Area} \): 225 \( \text{m}^2 \) \( \text{Surface Area} \): 345 \( \text{m}^2 \)

15. a right equilateral triangular prism with base edge length 8 ft and height 14 ft

Find the lateral area and surface area of each right cylinder. Give your answers in terms of \( \pi \).

16. \( \text{Lateral Area} \): 110 \( \pi \) \( \text{in}^2 \) \( \text{Surface Area} \): 176 \( \pi \) \( \text{in}^2 \)

17. \( \text{Lateral Area} \): 20 \( \pi \) \( \text{cm}^2 \) \( \text{Surface Area} \): 55 \( \pi \) \( \text{cm}^2 \)

18. a cylinder with base circumference \( 16\pi \) yd and a height equal to 3 times the radius

Multi-Step Find the surface area of each composite figure. Round to the nearest tenth.

19. \( \text{Surface Area} \): 230 \( \text{cm}^2 \)

20. \( \text{Surface Area} \): 30 \( \text{ft}^2 \)

Describe the effect of each change on the surface area of the given figure.

21. The dimensions are tripled.

22. The dimensions are doubled.

23. Biology Plant cells are shaped approximately like a right rectangular prism. Each cell absorbs oxygen and nutrients through its surface. Which cell can be expected to absorb at a greater rate? (Hint: 1 \( \mu \text{m} \) = 1 micrometer = 0.000001 meter)
24. Find the height of a right cylinder with surface area $160\pi \text{ ft}^2$ and radius 5 ft.

25. Find the height of a right rectangular prism with surface area $286 \text{ m}^2$, length 10 m, and width 8 m.

26. Find the height of a right regular hexagonal prism with lateral area $1368 \text{ m}^2$ and base edge length 12 m.

27. Find the surface area of the right triangular prism with vertices at $(0, 0, 0)$, $(5, 0, 0)$, $(0, 2, 0)$, $(0, 0, 9)$, $(5, 0, 9)$, and $(0, 2, 9)$.

The dimensions of various coins are given in the table. Find the surface area of each coin. Round to the nearest hundredth.

<table>
<thead>
<tr>
<th>Coin</th>
<th>Diameter (mm)</th>
<th>Thickness (mm)</th>
<th>Surface Area (mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>28. Penny</td>
<td>19.05</td>
<td>1.55</td>
<td></td>
</tr>
<tr>
<td>29. Nickel</td>
<td>21.21</td>
<td>1.95</td>
<td></td>
</tr>
<tr>
<td>30. Dime</td>
<td>17.91</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>31. Quarter</td>
<td>24.26</td>
<td>1.75</td>
<td></td>
</tr>
</tbody>
</table>

32. How can the edge lengths of a rectangular prism be changed so that the surface area is multiplied by 9?

33. How can the radius and height of a cylinder be changed so that the surface area is multiplied by $\frac{1}{4}$?

34. Landscaping Ingrid is building a shelter to protect her plants from freezing. She is planning to stretch plastic sheeting over the top and the ends of a frame. Which of the frames shown will require more plastic?

35. Critical Thinking If the length of the measurements of the net are correct to the nearest tenth of a centimeter, what is the maximum error in the surface area?

36. Write About It Explain how to use the net of a three-dimensional figure to find its surface area.

37. This problem will prepare you for the Multi-Step Test Prep on page 724.

A juice container is a square prism with base edge length 4 in. When an 8 in. straw is inserted into the container as shown, exactly 1 in. remains outside the container.

a. Find $AB$ and $BC$.

b. What is the height $AC$ of the container to the nearest tenth?

c. Use your result from part b to find how much material is required to manufacture the container. Round to the nearest tenth.
38. Measure the dimensions of the net of a cylinder to the nearest millimeter. Which is closest to the surface area of the cylinder?
   \[\begin{array}{ccc}
   \text{A} & 35.8 \text{ cm}^2 & \text{C} & 16.0 \text{ cm}^2 \\
   \text{B} & 18.8 \text{ cm}^2 & \text{D} & 13.2 \text{ cm}^2 \\
   \end{array}\]

39. The base of a triangular prism is an equilateral triangle with a perimeter of 24 inches. If the height of the prism is 5 inches, find the lateral area.
   \[\begin{array}{ccc}
   \text{F} & 120 \text{ in}^2 & \text{H} & 40 \text{ in}^2 \\
   \text{G} & 60 \text{ in}^2 & \text{I} & 360 \text{ in}^2 \\
   \end{array}\]

40. **Gridded Response** Find the surface area in square inches of a cylinder with a radius of 6 inches and a height of 5 inches. Use 3.14 for \(\pi\) and round your answer to the nearest tenth.

**CHALLENGE AND EXTEND**

41. A cylinder has a radius of 8 cm and a height of 3 cm. Find the height of another cylinder that has a radius of 4 cm and the same surface area as the first cylinder.

42. If one gallon of paint covers 250 square feet, how many gallons of paint will be needed to cover the shed, not including the roof? If a gallon of paint costs $25, about how much will it cost to paint the walls of the shed?

43. The lateral area of a right rectangular prism is 144 \(\text{cm}^2\). Its length is three times its width, and its height is twice its width. Find its surface area.

**SPIRAL REVIEW**

44. Rebecca’s car can travel 250 miles on one tank of gas. Rebecca has traveled 154 miles. Write an inequality that models \(m\), the number of miles farther Rebecca can travel on the tank of gas. (Previous course)

45. Blood sugar is a measure of the number of milligrams of glucose in a deciliter of blood (mg/dL). Normal fasting blood sugar levels are above 70 mg/dL and below 110 mg/dL. Write an inequality that models \(s\), the blood sugar level of a normal patient. (Previous course)

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree. (Lesson 8-5)

46. \(BC\)  
47. \(m\angle ABC\)

Draw the top, left, and right views of each object. Assume there are no hidden cubes. (Lesson 10-2)

48.  
49.  
50.
Model Right and Oblique Cylinders

In Lesson 10-4, you learned the difference between right and oblique cylinders. In this lab, you will make models of right and oblique cylinders.

Use with Lesson 10-4

Activity 1

1. Use a compass to draw at least 10 circles with a radius of 3 cm each on cardboard and then cut them out.

2. Poke a hole through the center of each circle.

3. Unbend a paper clip part way and push it through the center of each circle to model a cylinder. The stack of cardboard circles can be held straight to model a right cylinder or tilted to model an oblique cylinder.

Try This

1. On each cardboard model, use string or a rubber band to outline a cross section that is parallel to the base of the cylinder. What shape is each cross section?

2. Use string or a rubber band to outline a cross section of the cardboard model of the oblique cylinder that is perpendicular to the lateral surface. What shape is the cross section?

Activity 2

1. Roll a piece of paper to make a right cylinder. Tape the edges.

2. Cut along the bottom and top to approximate an oblique cylinder.

3. Untape the edge and unroll the paper. What does the net for an oblique cylinder look like?

Try This

3. Cut off the curved part of the net you created in Activity 2 and translate it to the opposite side to form a rectangle. How do the side lengths of the rectangle relate to the dimensions of the cylinder? Estimate the lateral area and surface area of the oblique cylinder.
10-5 Surface Area of Pyramids and Cones

Objectives
Learn and apply the formula for the surface area of a pyramid.
Learn and apply the formula for the surface area of a cone.

Vocabulary
vertex of a pyramid
regular pyramid
slant height of a regular pyramid
altitude of a pyramid
vertex of a cone
right cone
oblique cone
slant height of a right cone
altitude of a cone

Why learn this?
A speaker uses part of the lateral surface of a cone to produce sound. Speaker cones are usually made of paper, plastic, or metal. (See Example 5.)

The vertex of a pyramid is the point opposite the base of the pyramid. The base of a regular pyramid is a regular polygon, and the lateral faces are congruent isosceles triangles. The slant height of a regular pyramid is the distance from the vertex to the midpoint of an edge of the base. The altitude of a pyramid is the perpendicular segment from the vertex to the plane of the base.

The lateral faces of a regular pyramid can be arranged to cover half of a rectangle with a height equal to the slant height of the pyramid. The width of the rectangle is equal to the base perimeter of the pyramid.

Lateral and Surface Area of a Regular Pyramid

The lateral area of a regular pyramid with perimeter \( P \) and slant height \( \ell \) is \( L = \frac{1}{2}P\ell \).

The surface area of a regular pyramid with lateral area \( L \) and base area \( B \) is \( S = L + B \), or \( S = \frac{1}{2}P\ell + B \).

Example 1 Finding Lateral Area and Surface Area of Pyramids

Find the lateral area and surface area of each pyramid.

A regular square pyramid with base edge length 5 in. and slant height 9 in.

Lateral area of a regular pyramid

\[
P = 4(5) = 20 \text{ in.}
\]

Surface area of a regular pyramid

\[
B = S^2 = 25 \text{ in}^2
\]

10-5 Surface Area of Pyramids and Cones 689
Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth.

**Step 1** Find the base perimeter and apothem.

The base perimeter is $6(4) = 24$ m.

The apothem is $2\sqrt{3}$ m, so the base area is $\frac{1}{2}aP = \frac{1}{2}(2\sqrt{3})(24) = 24\sqrt{3}$ m².

**Step 2** Find the lateral area.

$L = \frac{1}{2} P\ell$  
$L = \frac{1}{2}(24)(7) = 84$ m²  
*Substitute 24 for $P$ and 7 for $\ell$.*

**Step 3** Find the surface area.

$S = \frac{1}{2}P\ell + B$  
$S = 84 + 24\sqrt{3} \approx 125.6$ cm²  
*Substitute $24\sqrt{3}$ for $B$.  

1. Find the lateral area and surface area of a regular triangular pyramid with base edge length 6 ft and slant height 10 ft.

The **vertex of a cone** is the point opposite the base. The **axis of a cone** is the segment with endpoints at the vertex and the center of the base. The axis of a **right cone** is perpendicular to the base. The axis of an **oblique cone** is not perpendicular to the base.

The **slant height of a right cone** is the distance from the vertex of a right cone to a point on the edge of the base. The **altitude of a cone** is a perpendicular segment from the vertex of the cone to the plane of the base.

**Lateral and Surface Area of a Right Cone**

The lateral area of a right cone with radius $r$ and slant height $\ell$ is $L = \pi r\ell$.

The surface area of a right cone with lateral area $L$ and base area $B$ is $S = L + B$, or $S = \pi\ell + \pi r^2$.  

690  Chapter 10 Spatial Reasoning
**Finding Lateral Area and Surface Area of Right Cones**

Find the lateral area and surface area of each cone. Give your answers in terms of $\pi$.

**A**  
A right cone with radius 2 m and slant height 3 m

$L = \pi r\ell$  
$L = \pi (2)(3) = 6\pi$ m$^2$  
$L = 6\pi$ m$^2$  

$S = \pi r\ell + \pi r^2$  
$S = 6\pi + \pi (2)^2 = 10\pi$ m$^2$

**B**

Step 1 Use the Pythagorean Theorem to find $\ell$.

$$\ell = \sqrt{5^2 + 12^2} = 13$$ ft

Step 2 Find the lateral area and surface area.

$L = \pi r\ell$  
$L = \pi (5)(13) = 65\pi$ ft$^2$  
$L = 65\pi$ ft$^2$

$S = \pi r\ell + \pi r^2$  
$S = 65\pi + \pi (5)^2 = 90\pi$ ft$^2$  

**Check It Out**

2. Find the lateral area and surface area of the right cone.

**Exploring Effects of Changing Dimensions**

The radius and slant height of the right cone are tripled. Describe the effect on the surface area.

original dimensions:  
$S = \pi r\ell + \pi r^2$  
$S = \pi (3)(5) + \pi (3)^2$  
$S = 24\pi$ cm$^2$

radius and slant height tripled:  
$S = \pi r\ell + \pi r^2$  
$S = \pi (9)(15) + \pi (9)^2$  
$S = 216\pi$ cm$^2$

Notice that $216\pi = 9(24\pi)$. If the radius and slant height are tripled, the surface area is multiplied by $3^2$, or 9.

**Check It Out**

3. The base edge length and slant height of the regular square pyramid are both multiplied by $\frac{2}{3}$. Describe the effect on the surface area.
**Example 4** Finding Surface Area of Composite Three-Dimensional Figures

Find the surface area of the composite figure.

![Diagram of a cone and a cylinder with dimensions]

The height of the cone is $90 - 45 = 45$ cm. By the Pythagorean Theorem, 

$$\ell = \sqrt{28^2 + 45^2} = 53$$ cm. The lateral area of the cone is 

$$L = \pi \ell = \pi (28)(53) = 1484 \pi \text{ cm}^2.$$ 

The lateral area of the cylinder is 

$$L = 2\pi rh = 2\pi (28)(45) = 2520 \pi \text{ cm}^2.$$ 

The base area is 

$$B = \pi r^2 = \pi (28)^2 = 784 \pi \text{ cm}^2.$$ 

$S = (\text{cone lateral area}) + (\text{cylinder lateral area}) + (\text{base area})$ 

$$= 2520 \pi + 784 \pi + 1484 \pi = 4788 \pi \text{ cm}^2.$$ 

4. Find the surface area of the composite figure.

**Example 5** Electronics Application

Tim is replacing the paper cone of an antique speaker. He measured the existing cone and created the pattern for the lateral surface from a large circle.

**What is the diameter of the cone?**

The radius of the large circle used to create the pattern is the slant height of the cone.

The area of the pattern is the lateral area of the cone. The area of the pattern is also $\frac{3}{4}$ of the area of the large circle, so $\pi r \ell = \frac{3}{4} \pi r^2$.

$$\pi r(10) = \frac{3}{4} \pi (10)^2$$ 

Substitute 10 for $\ell$, the slant height of the cone and the radius of the large circle.

$$r = 7.5 \text{ in.}$$ 

Solve for $r$.

The diameter of the cone is $2(7.5) = 15$ in.

5. **What if...?** If the radius of the large circle were 12 in., what would be the radius of the cone?

**Think and Discuss**

1. Explain why the lateral area of a regular pyramid is $\frac{1}{2}$ the base perimeter times the slant height.
2. In a right cone, which is greater, the height or the slant height? Explain.
3. **Get Organized** Copy and complete the graphic organizer. In each box, write the name of the part of the cone.
1. **Vocabulary**  Describe the endpoints of an axis of a cone.

Find the lateral area and surface area of each regular pyramid.

2. [Diagram of a pyramid with dimensions: base edge 12 cm, height 8 cm]

3. [Diagram of a pyramid with dimensions: base edge 15 ft, height 16 ft]

4. a regular triangular pyramid with base edge length 15 in. and slant height 20 in.

Find the lateral area and surface area of each right cone. Give your answers in terms of \( \pi \).

5. [Diagram of a cone with dimensions: base radius 24 in., slant height 25 in.]

6. [Diagram of a cone with dimensions: base radius 22 m, slant height 14 m]

7. a cone with base area \( 36\pi \) ft\(^2\) and slant height 8 ft

Describe the effect of each change on the surface area of the given figure.

8. The dimensions are cut in half.

9. The dimensions are tripled.

Find the surface area of each composite figure.

10. [Diagram of a composite figure with dimensions: height 15 ft, base 8 ft, and side 18 ft]

11. [Diagram of a composite figure with dimensions: height 26 m, base 12 m, and side 32 m]

12. **Crafts**  Anna is making a birthday hat from a pattern that is \( \frac{3}{4} \) of a circle of colored paper. If Anna's head is 7 inches in diameter, will the hat fit her? Explain.
PRACTICE AND PROBLEM SOLVING

Find the lateral area and surface area of each regular pyramid.

13. [Diagram: Regular pyramid with base edge length 4 ft and slant height 6 ft]
14. [Diagram: Regular pyramid with base edge length 25 cm and slant height 40 cm]

15. a regular hexagonal pyramid with base edge length 7 ft and slant height 15 ft

Find the lateral area and surface area of each right cone. Give your answers in terms of \( \pi \).

16. [Diagram: Right cone with radius 23 cm and slant height 24 cm]
17. [Diagram: Right cone with radius 35 in. and slant height 24 in.]

18. a cone with radius 8 m and height that is 1 m less than twice the radius

Describe the effect of each change on the surface area of the given figure.

19. The dimensions are divided by 3.
20. The dimensions are doubled.

Find the surface area of each composite figure.

21. [Diagram: Composite figure with base dimensions 24 in. x 7 in. and height 17 in.]
22. [Diagram: Composite figure with base dimensions 15 cm x 9 cm and height 19 cm]

23. It is a tradition in England to celebrate May 1st by hanging cone-shaped baskets of flowers on neighbors’ door handles. Addy is making a basket from a piece of paper that is a semicircle with diameter 12 in. What is the diameter of the basket?

Find the surface area of each figure.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Base Area</th>
<th>Slant Height</th>
<th>Surface Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>24. Regular square pyramid</td>
<td>36 cm²</td>
<td>5 cm</td>
<td></td>
</tr>
<tr>
<td>25. Regular triangular pyramid</td>
<td>( \sqrt{3} ) m²</td>
<td>( \sqrt{3} ) m</td>
<td></td>
</tr>
<tr>
<td>26. Right cone</td>
<td>16( \pi ) in²</td>
<td>7 in.</td>
<td></td>
</tr>
<tr>
<td>27. Right cone</td>
<td>( \pi ) ft²</td>
<td>2 ft</td>
<td></td>
</tr>
</tbody>
</table>
28. This problem will prepare you for the Multi-Step Test Prep on page 724. A juice container is a regular square pyramid with the dimensions shown.
   a. Find the surface area of the container to the nearest tenth.
   b. The manufacturer decides to make a container in the shape of a right cone that requires the same amount of material. The base diameter must be 9 cm. Find the slant height of the container to the nearest tenth.

29. Find the radius of a right cone with slant height 21 m and surface area $232\pi \text{ m}^2$.
30. Find the slant height of a regular square pyramid with base perimeter 32 ft and surface area 256 ft$^2$.
31. Find the base perimeter of a regular hexagonal pyramid with slant height 10 cm and lateral area 120 cm$^2$.
32. Find the surface area of a right cone with a slant height of 25 units that has its base centered at $(0, 0, 0)$ and its vertex at $(0, 0, 7)$.

Find the surface area of each composite figure.
33. 
34. 

35. **Architecture** The Pyramid Arena in Memphis, Tennessee, is a square pyramid with base edge lengths of 200 yd and a height of 32 stories. Estimate the area of the glass on the sides of the pyramid. *(Hint: 1 story $\approx 10$ ft)*

36. **Critical Thinking** Explain why the slant height of a regular square pyramid must be greater than half the base edge length.

37. **Write About It** Explain why slant height is not defined for an oblique cone.

38. Which expressions represent the surface area of the regular square pyramid?
   I. $\frac{t^2}{16} + \frac{ts}{2}$  
   II. $\frac{t^2}{16} + \frac{tl}{2}$  
   III. $\frac{t}{2} (8 + \ell)$
   A. I only  
   B. II only  
   C. I and II  
   D. II and III

39. A regular square pyramid has a slant height of 18 cm and a lateral area of 216 cm$^2$. What is the surface area?
   F. $252 \text{ cm}^2$  
   G. $234 \text{ cm}^2$  
   H. $225 \text{ cm}^2$  
   J. $240 \text{ cm}^2$

40. What is the lateral area of the cone?
   A. $360\pi \text{ cm}^2$  
   B. $369\pi \text{ cm}^2$  
   C. $450\pi \text{ cm}^2$  
   D. $1640\pi \text{ cm}^2$
**CHALLENGE AND EXTEND**

41. A frustum of a cone is a part of the cone with two parallel bases. The height of the frustum of the cone is half the height of the original cone.
   a. Find the surface area of the original cone.
   b. Find the lateral area of the top of the cone.
   c. Find the area of the top base of the frustum.
   d. Use your results from parts a, b, and c to find the surface area of the frustum of the cone.

42. A frustum of a pyramid is a part of the pyramid with two parallel bases. The lateral faces of the frustum are trapezoids. Use the area formula for a trapezoid to derive a formula for the lateral area of a frustum of a regular square pyramid with base edge lengths $b_1$ and $b_2$ and slant height $\ell$.

43. Use the net to derive the formula for the lateral area of a right cone with radius $r$ and slant height $\ell$.
   a. The length of the curved edge of the lateral surface must equal the circumference of the base. Find the circumference $c$ of the base in terms of $r$.
   b. The lateral surface is part of a larger circle. Find the circumference $C$ of the larger circle.
   c. The lateral surface area is $\frac{c}{C}$ times the area of the larger circle. Use your results from parts a and b to find $\frac{c}{C}$.
   d. Find the area of the larger circle. Use your result and the result from part c to find the lateral area $L$.

**SPIRAL REVIEW**

State whether the following can be described by a linear function. *(Previous course)*

44. the surface area of a right circular cone with height $h$ and radius $r$

45. the perimeter of a rectangle with a height $h$ that is twice as large as its width $w$

46. the area of a circle with radius $r$

A point is chosen randomly in $ACEF$. Find the probability of each event. Round to the nearest hundredth. *(Lesson 9-6)*

47. The point is in $\triangle BDG$.

48. The point is in $\odot H$.

49. The point is in the shaded region.

Find the surface area of each right prism or right cylinder. Round your answer to the nearest tenth. *(Lesson 10-4)*

50. 15 in. 17 in. 10 in.

51. 8 cm 10 cm 15 cm

52. 2 cm 3 cm
Objectives
Learn and apply the formula for the volume of a prism.
Learn and apply the formula for the volume of a cylinder.

Vocabulary
volume

Who uses this?
Marine biologists must ensure that aquariums are large enough to accommodate the number of fish inside them. (See Example 2.)

The volume of a three-dimensional figure is the number of nonoverlapping unit cubes of a given size that will exactly fill the interior.

Cavalieri’s principle says that if two three-dimensional figures have the same height and have the same cross-sectional area at every level, they have the same volume.

Volume of a Prism

The volume of a prism with base area \( B \) and height \( h \) is \( V = Bh \).

Example 1: Finding Volumes of Prisms

Find the volume of each prism. Round to the nearest tenth, if necessary.

A right rectangular prism with length \( \ell \), width \( w \), and height \( h \) is \( V = \ell wh \).

The volume of a cube with edge length \( s \) is \( V = s^3 \).

A cube built out of 27 unit cubes has a volume of 27 cubic units.

A right prism and an oblique prism with the same base and height have the same volume.

Example 1

Find the volume of each prism. Round to the nearest tenth, if necessary.

A

Volume of a right rectangular prism

Substitute 10 for \( \ell \), 12 for \( w \), and 8 for \( h \).

\[ V = \ell wh \]
\[ = (10)(12)(8) = 960 \text{ cm}^3 \]

B

Volume of a cube

Substitute 10 for \( s \).

\[ V = s^3 \]
\[ = 10^3 = 1000 \text{ cm}^3 \]
Find the volume of each prism. Round to the nearest tenth, if necessary.

**C** a right regular pentagonal prism with base edge length 5 m and height 7 m

**Step 1** Find the apothem \( a \) of the base. First draw a right triangle on one base as shown. The measure of the angle with its vertex at the center is \( \frac{360°}{10} = 36° \).

\[
\tan 36° = \frac{2.5}{a} \quad \text{The leg of the triangle is half the side length, or 2.5 m.}
\]

\[
a = \frac{2.5}{\tan 36°} \quad \text{Solve for } a.
\]

**Step 2** Use the value of \( a \) to find the base area.

\[
B = \frac{1}{2} aP = \frac{1}{2} \left( \frac{2.5}{\tan 36°} \right) (25) = \frac{31.25}{\tan 36°} \quad P = 5(5) = 25 \text{ m}
\]

**Step 3** Use the base area to find the volume.

\[
V = Bh = \frac{31.25}{\tan 36°} \cdot 7 \approx 301.1 \text{ m}^3
\]

1. Find the volume of a triangular prism with a height of 9 yd whose base is a right triangle with legs 7 yd and 5 yd long.

**Example 2** Marine Biology Application

The aquarium at the right is a rectangular prism. Estimate the volume of the water in the aquarium in gallons. The density of water is about 8.33 pounds per gallon. Estimate the weight of the water in pounds. (Hint: 1 gallon \( \approx 0.134 \text{ ft}^3 \))

**Step 1** Find the volume of the aquarium in cubic feet.

\[
V = \ellwh = (120)(60)(8) = 57,600 \text{ ft}^3
\]

**Step 2** Use the conversion factor \( \frac{1 \text{ gallon}}{0.134 \text{ ft}^3} \) to estimate the volume in gallons.

\[
57,600 \text{ ft}^3 \cdot \frac{1 \text{ gallon}}{0.134 \text{ ft}^3} \approx 429,851 \text{ gallons}
\]

**Step 3** Use the conversion factor \( \frac{8.33 \text{ pounds}}{1 \text{ gallon}} \) to estimate the weight of the water.

\[
429,851 \text{ gallons} \cdot \frac{8.33 \text{ pounds}}{1 \text{ gallon}} \approx 3,580,659 \text{ pounds}
\]

The aquarium holds about 429,851 gallons. The water in the aquarium weighs about 3,580,659 pounds.

2. What if…? Estimate the volume in gallons and the weight of the water in the aquarium above if the height were doubled.
Cavalieri’s principle also relates to cylinders. The two stacks have the same number of CDs, so they have the same volume.

Volume of a Cylinder

The volume of a cylinder with base area $B$, radius $r$, and height $h$ is $V = Bh$, or $V = \pi r^2 h$.

Example 3: Finding Volumes of Cylinders

Find the volume of each cylinder. Give your answers both in terms of $\pi$ and rounded to the nearest tenth.

A

Find the volume of this cylinder.

$V = \pi r^2 h$

$V = \pi (8)^2 (12)$

$V = 768 \pi$ cm$^3 \approx 2412.7$ cm$^3$

B

A cylinder with a base area of $36\pi$ in$^2$ and a height equal to twice the radius.

Step 1 Use the base area to find the radius.

$$\pi r^2 = 36\pi$$

$r = 6$

Step 2 Use the radius to find the height. The height is equal to twice the radius.

$h = 2r$

$h = 2(6) = 12$ cm

Step 3 Use the radius and height to find the volume.

$V = \pi r^2 h$

$V = \pi (6)^2 (12) = 432\pi$ in$^3$

$V \approx 1357.2$ in$^3$

3. Find the volume of a cylinder with a diameter of 16 in. and a height of 17 in. Give your answer both in terms of $\pi$ and rounded to the nearest tenth.
**Example 4**

**Exploring Effects of Changing Dimensions**

The radius and height of the cylinder are multiplied by $\frac{1}{2}$. Describe the effect on the volume.

Original dimensions:

\[
V = \pi r^2 h
= \pi (6)^2 (12)
= 432\pi \text{ m}^3
\]

Radius and height multiplied by $\frac{1}{2}$:

\[
V = \pi \left(\frac{1}{2}r\right)^2 \left(\frac{1}{2}h\right)
= \pi \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right) (6)^2 (12)
= \pi \left(\frac{1}{4}\right) \left(\frac{1}{2}\right) (6)^2 (12)
= \frac{1}{8} \pi (36)(12)
= 54\pi \text{ m}^3
\]

Notice that $54\pi = \frac{1}{8}(432\pi)$. If the radius and height are multiplied by $\frac{1}{2}$, the volume is multiplied by $\left(\frac{1}{2}\right)^3$, or $\frac{1}{8}$.

**Check it Out!**

4. The length, width, and height of the prism are doubled. Describe the effect on the volume.

**Example 5**

**Finding Volumes of Composite Three-Dimensional Figures**

Find the volume of the composite figure. Round to the nearest tenth.

The base area of the prism is $B = \frac{1}{2}(6)(8) = 24 \text{ m}^2$.

The volume of the prism is $V = Bh = 24(9) = 216 \text{ m}^3$.

The cylinder’s diameter equals the hypotenuse of the prism’s base, 10 m. So the radius is 5 m.

The volume of the cylinder is $V = \pi r^2 h = \pi (5)^2 (5) = 125\pi \text{ m}^3$.

The total volume of the figure is the sum of the volumes. $V = 216 + 125\pi \approx 608.7 \text{ m}^3$.

**Check it Out!**

5. Find the volume of the composite figure. Round to the nearest tenth.

**Think and Discuss**

1. Compare the formula for the volume of a prism with the formula for the volume of a cylinder.

2. Explain how Cavalieri’s principle relates to the formula for the volume of an oblique prism.

3. **Get Organized** Copy and complete the graphic organizer. In each box, write the formula for the volume.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Volume</th>
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<tbody>
<tr>
<td>Prism</td>
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<tr>
<td>Cube</td>
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<tr>
<td>Cylinder</td>
<td></td>
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</tbody>
</table>
1. **Vocabulary** In a right cylinder, the *altitude* is ___?___ the axis. (*longer than, shorter than, or the same length as*)

Find the volume of each prism.

2. 

3. 

4. a cube with edge length 8 ft

5. **Food** The world’s largest ice cream cake, built in New York City on May 25, 2004, was approximately a 19 ft by 9 ft by 2 ft rectangular prism. Estimate the volume of the ice cream cake in gallons. If the density of the ice cream was 4.73 pounds per gallon, estimate the weight of the cake. (*Hint: 1 gallon ≈ 0.134 cubic feet*)

Find the volume of each cylinder. Give your answers both in terms of \(\pi\) and rounded to the nearest tenth.

6. 

7. 

8. a cylinder with base area \(25\pi\) cm\(^2\) and height 3 cm more than the radius

Describe the effect of each change on the volume of the given figure.

9. The dimensions are multiplied by \(\frac{1}{4}\)

10. The dimensions are tripled.

Find the volume of each composite figure. Round to the nearest tenth.

11. 

12. 

Find the volume of each prism.

13. \[
\text{Volume} = 15 \times 12 \times 9 = 1620 \text{yd}^3
\]

14. \[
\text{Volume} = 72 \times 15 = 1080 \text{m}^3
\]

15. A square prism with a base area of 49 $\text{ft}^2$ and a height 2 ft less than the base edge length.

16. **Landscaping** Colin is buying dirt to fill a garden bed that is a 9 ft by 16 ft rectangle. If he wants to fill it to a depth of 4 in., how many cubic yards of dirt does he need? If dirt costs $25 per yd$^3$, how much will the project cost? (Hint: 1 yd$^3$ = 27 ft$^3$)

Find the volume of each cylinder. Give your answers both in terms of $\pi$ and rounded to the nearest tenth.

17. \[
\text{Volume} = \pi \times 14 \times 9 = 126\pi \text{cm}^3
\]

18. \[
\text{Volume} = \frac{1}{2} \times 6 \times 3 = 9 \text{in}^3
\]

19. A cylinder with base area $24\pi \text{cm}^2$ and height 16 cm

Describe the effect of each change on the volume of the given figure.

20. The dimensions are multiplied by 5.

21. The dimensions are multiplied by $\frac{3}{5}$.

Find the volume of each composite figure.

22.

23.

24. One cup is equal to 14.4375 in$^3$. If a 1 c cylindrical measuring cup has a radius of 2 in., what is its height? If the radius is 1.5 in., what is its height?

25. **Food** A cake is a cylinder with a diameter of 10 in. and a height of 3 in. For a party, a coin has been mixed into the batter and baked inside the cake. The person who gets the piece with the coin wins a prize.

   a. Find the volume of the cake. Round to the nearest tenth.

   b. **Probability** Keka gets a piece of cake that is a right rectangular prism with a 3 in. by 1 in. base. What is the probability that the coin is in her piece? Round to the nearest hundredth.
26. This problem will prepare you for the Multi-Step Test Prep on page 724.
A cylindrical juice container with a 3 in. diameter has a hole for a straw that is 1 in. from the side. Up to 5 in. of a straw can be inserted.
   a. Find the height \( h \) of the container to the nearest tenth.
   b. Find the volume of the container to the nearest tenth.
   c. How many ounces of juice does the container hold?  
      (Hint: 1 in\(^3\) \approx 0.55 oz)

27. Find the height of a rectangular prism with length 5 ft, width 9 ft, and volume 495 ft\(^3\).
28. Find the area of the base of a rectangular prism with volume 360 in\(^3\) and height 9 in.
29. Find the volume of a cylinder with surface area \( 210\pi \text{ m}^2 \) and height 8 m.
30. Find the volume of a rectangular prism with vertices \((0, 0, 0)\), \((0, 3, 0)\), \((7, 0, 0)\), \((7, 3, 0)\), \((0, 0, 6)\), \((0, 3, 6)\), \((7, 0, 6)\), and \((7, 3, 6)\).
31. You can use displacement to find the volume of an irregular object, such as a stone. Suppose the tank shown is filled with water to a depth of 8 in. A stone is placed in the tank so that it is completely covered, causing the water level to rise by 2 in. Find the volume of the stone.
32. Food A 1 in. cube of cheese is one serving. How many servings are in a 4 in. by 4 in. by \( \frac{1}{4} \) in. slice?
33. History In 1919, a cylindrical tank containing molasses burst and flooded the city of Boston, Massachusetts. The tank had a 90 ft diameter and a height of 52 ft. How many gallons of molasses were in the tank?  
   (Hint: 1 gal \approx 0.134 ft\(^3\))
34. Meteorology If 3 in. of rain fall on the property shown, what is the volume in cubic feet? In gallons? The density of water is 8.33 pounds per gallon. What is the weight of the rain in pounds?  
   (Hint: 1 gal \approx 0.134 ft\(^3\))
35. Critical Thinking The dimensions of a prism with volume \( V \) and surface area \( S \) are multiplied by a scale factor of \( k \) to form a similar prism. Make a conjecture about the ratio of the surface area of the new prism to its volume. Test your conjecture using a cube with an edge length of 1 and a scale factor of 2.
36. Write About It How can you change the edge length of a cube so that its volume is doubled?
37. Abigail has a cylindrical candle mold with the dimensions shown. If Abigail has a rectangular block of wax measuring 15 cm by 12 cm by 18 cm, about how many candles can she make after melting the block of wax?
   \[ \text{A} \quad 14 \quad \text{B} \quad 31 \quad \text{C} \quad 35 \quad \text{D} \quad 76 \]
38. A 96-inch piece of wire was cut into equal segments that were then connected to form the edges of a cube. What is the volume of the cube?
- $512	ext{ in}^3$
- $576	ext{ in}^3$
- $729	ext{ in}^3$
- $1728	ext{ in}^3$

39. One juice container is a rectangular prism with a height of 9 in. and a 3 in. by 3 in. square base. Another juice container is a cylinder with a radius of 1.75 in. and a height of 9 in. Which best describes the relationship between the two containers?
- A. The prism has the greater volume.
- B. The cylinder has the greater volume.
- C. The volumes are equivalent.
- D. The volumes cannot be determined.

40. What is the volume of the three-dimensional object with the dimensions shown in the three views below?
- $160	ext{ cm}^3$
- $240	ext{ cm}^3$
- $840	ext{ cm}^3$
- $1000	ext{ cm}^3$

**CHALLENGE AND EXTEND**

**Algebra** Find the volume of each three-dimensional figure in terms of $x$.

41. 

42. 

43. 

44. The volume in cubic units of a cylinder is equal to its surface area in square units. Prove that the radius and height must both be greater than 2.

**SPIRAL REVIEW**

45. Marcy, Rachel, and Tina went bowling. Marcy bowled 100 less than twice Rachel’s score. Tina bowled 40 more than Rachel’s score. Rachel bowled a higher score than Marcy. What is the greatest score that Tina could have bowled? *(Previous course)*

46. Max can type 40 words per minute. He estimates that his term paper contains about 5000 words, and he takes a 15-minute break for every 45 minutes of typing. About how much time will it take Max to type his term paper? *(Previous course)*

**ABCD is a parallelogram. Find each measure.** *(Lesson 6-2)*

47. $m\angle ABC$

48. $BC$

49. $AB$

Find the surface area of each figure. Round to the nearest tenth. *(Lesson 10-5)*

50. a square pyramid with slant height 10 in. and base edge length 8 in.

51. a regular pentagonal pyramid with slant height 8 cm and base edge length 6 cm

52. a right cone with slant height 2 ft and a base with circumference of $\pi$ ft
Objectives
Learn and apply the formula for the volume of a pyramid.
Learn and apply the formula for the volume of a cone.

Who uses this?
The builders of the Rainforest Pyramid in Galveston, Texas, needed to calculate the volume of the pyramid to plan the climate control system. (See Example 2.)

The volume of a pyramid is related to the volume of a prism with the same base and height. The relationship can be verified by dividing a cube into three congruent square pyramids, as shown.

The square pyramids are congruent, so they have the same volume. The volume of each pyramid is one third the volume of the cube.

Volume of a Pyramid
The volume of a pyramid with base area \( B \) and height \( h \) is \( V = \frac{1}{3}Bh \).

Example 1
Finding Volumes of Pyramids
Find the volume of each pyramid.

A a rectangular pyramid with length 7 ft, width 9 ft, and height 12 ft
\[ V = \frac{1}{3}Bh = \frac{1}{3}(7 \cdot 9)(12) = 252 \text{ ft}^3 \]

B the square pyramid
The base is a square with a side length of 4 in., and the height is 6 in.
\[ V = \frac{1}{3}Bh = \frac{1}{3}(4^2)(6) = 32 \text{ in}^3 \]
Find the volume of the pyramid. C the trapezoidal pyramid with base $ABCD$, where $AB \parallel CD$ and $AE \perp$ plane $ABC$

**Step 1** Find the area of the base.

$B = \frac{1}{2}(b_1 + b_2)h \quad \text{Area of a trapezoid}$

$= \frac{1}{2}(9 + 18)6 \quad \text{Substitute 9 for } b_1, 18 \text{ for } b_2, \text{ and 6 for } h.$

$= 81 \text{ m}^2 \quad \text{Simplify.}$

**Step 2** Use the base area and the height to find the volume.
Because $AE \perp$ plane $ABC$, $AE$ is the altitude, so the height is equal to $AE$.

$V = \frac{1}{3}Bh \quad \text{Volume of a pyramid}$

$= \frac{1}{3}(81)(10) \quad \text{Substitute 81 for } B \text{ and 10 for } h.$

$= 270 \text{ m}^3$

1. Find the volume of a regular hexagonal pyramid with a base edge length of 2 cm and a height equal to the area of the base.

**Example 2**

**Architecture Application**

The Rainforest Pyramid in Galveston, Texas, is a square pyramid with a base area of about 1 acre and a height of 10 stories. Estimate the volume in cubic yards and in cubic feet. *(Hint: 1 acre = 4840 yd$^2$, 1 story $\approx$ 10 ft)*

The base is a square with an area of about 4840 yd$^2$. The base edge length is $\sqrt{4840} \approx 70$ yd. The height is about $10(10) = 100$ ft, or about 33 yd.

First find the volume in cubic yards.

$V = \frac{1}{3}Bh \quad \text{Volume of a regular pyramid}$

$= \frac{1}{3}(70^2)(33) = 53,900 \text{ yd}^3 \quad \text{Substitute } 70^2 \text{ for } B \text{ and 33 for } h.$

Then convert your answer to find the volume in cubic feet. The volume of one cubic yard is $(3 \text{ ft})(3 \text{ ft})(3 \text{ ft}) = 27 \text{ ft}^3$. Use the conversion factor $\frac{27 \text{ ft}^3}{1 \text{ yd}^3}$ to find the volume in cubic feet.

$53,900 \text{ yd}^3 \cdot \frac{27 \text{ ft}^3}{1 \text{ yd}^3} \approx 1,455,300 \text{ ft}^3$

2. **What if…?** What would be the volume of the Rainforest Pyramid if the height were doubled?
The volume of a cone with base area $B$, radius $r$, and height $h$ is $V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2h$.

**Example 3**

**Finding Volumes of Cones**

Find the volume of each cone. Give your answers both in terms of $\pi$ and rounded to the nearest tenth.

**A**

a cone with radius 5 cm and height 12 cm

$V = \frac{1}{3}\pi r^2h$

Volume of a cone

$= \frac{1}{3}\pi(5)^2(12)$

Substitute 5 for $r$ and 12 for $h$.

$= 100\pi \text{ cm}^3 \approx 314.2 \text{ cm}^3$

Simplify.

**B**

a cone with a base circumference of $21\pi$ cm and a height 3 cm less than twice the radius

**Step 1** Use the circumference to find the radius.

$2\pi r = 21\pi$

Substitute $21\pi$ for $C$.

$r = 10.5 \text{ cm}$

Divide both sides by $2\pi$.

**Step 2** Use the radius to find the height.

$2(10.5) - 3 = 18$ cm

The height is 3 cm less than twice the radius.

**Step 3** Use the radius and height to find the volume.

$V = \frac{1}{3}\pi r^2h$

Volume of a cone

$= \frac{1}{3}\pi(10.5)^2(18)$

Substitute 10.5 for $r$ and 18 for $h$.

$= 661.5\pi \text{ cm}^3 \approx 2078.2 \text{ cm}^3$

Simplify.

**C**

**Step 1** Use the Pythagorean Theorem to find the height.

$7^2 + h^2 = 25^2$

$Pythagorean \ Theorem$

$h^2 = 576$

Subtract $7^2$ from both sides.

$h = 24$

Take the square root of both sides.

**Step 2** Use the radius and height to find the volume.

$V = \frac{1}{3}\pi r^2h$

Volume of a cone

$= \frac{1}{3}\pi(7)^2(24)$

Substitute 7 for $r$ and 24 for $h$.

$= 392\pi \text{ ft}^3 \approx 1231.5 \text{ ft}^3$

Simplify.

CHECK IT OUT!

3. Find the volume of the cone.
EXAMPEL 4 Exploring Effects of Changing Dimensions

The length, width, and height of the rectangular pyramid are multiplied by \( \frac{1}{4} \). Describe the effect on the volume.

original dimensions: \[ V = \frac{1}{3}Bh \]
\[ = \frac{1}{3}(24 \cdot 20)(20) \]
\[ = 3200 \text{ ft}^3 \]

length, width, and height multiplied by \( \frac{1}{4} \):
\[ V = \frac{1}{3}Bh \]
\[ = \frac{1}{3}(6 \cdot 5)(5) \]
\[ = 50 \text{ ft}^3 \]

Notice that \( 50 = \frac{1}{64}(3200) \). If the length, width, and height are multiplied by \( \frac{1}{4} \), the volume is multiplied by \( \left( \frac{1}{4} \right)^3 \), or \( \frac{1}{64} \).

EXAMPEL 5 Finding Volumes of Composite Three-Dimensional Figures

Find the volume of the composite figure. Round to the nearest tenth.

The volume of the cylinder is
\[ V = \pi r^2h = \pi (2)^2(2) = 8\pi \text{ in}^3. \]

The volume of the cone is
\[ V = \frac{1}{3}\pi r^2h = \frac{1}{3}\pi (2)^2(3) = 4\pi \text{ in}^3. \]

The volume of the composite figure is the sum of the volumes.
\[ V = 8\pi + 4\pi = 12\pi \approx 37.7 \text{ in}^3 \]

THINK AND DISCUSS

1. Explain how the volume of a pyramid is related to the volume of a prism with the same base and height.

2. GET ORGANIZED Copy and complete the graphic organizer.

<table>
<thead>
<tr>
<th>Volumes of Three-Dimensional Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formula</td>
</tr>
<tr>
<td>Shapes</td>
</tr>
<tr>
<td>Examples</td>
</tr>
</tbody>
</table>
1. **Vocabulary** The *altitude* of a pyramid is ____ to the base. (*perpendicular, parallel, or oblique*)

2. Find the volume of each pyramid. Round to the nearest tenth, if necessary.

3. a hexagonal pyramid with a base area of 25 ft² and a height of 9 ft

4. **Geology** A crystal is cut into the shape formed by two square pyramids joined at the base. Each pyramid has a base edge length of 5.7 mm and a height of 3 mm. What is the volume to the nearest cubic millimeter of the crystal?

5. Find the volume of each cone. Give your answers both in terms of π and rounded to the nearest tenth.

6. a cone with radius 12 m and height 20 m

7. Describe the effect of each change on the volume of the given figure.

9. The dimensions are tripled.

10. The dimensions are multiplied by \(\frac{1}{2}\).

11. Find the volume of each composite figure. Round to the nearest tenth, if necessary.
PRACTICE AND PROBLEM SOLVING

Find the volume of each pyramid. Round to the nearest tenth, if necessary.

13. \[
\text{Volume of a pyramid} = \frac{1}{3} \times \text{base area} \times \text{height} \\
= \frac{1}{3} \times (8 \times 6) \times 10 = \frac{1}{3} \times 48 \times 10 = 160 \\
\]

14. \[
\text{Volume of a pyramid} = \frac{1}{3} \times \text{base area} \times \text{height} \\
= \frac{1}{3} \times (12 \times 5) \times 13 \approx 265 \\
\]

15. A regular square pyramid with base edge length 12 ft and slant height 10 ft

16. Carpentry A roof that encloses an attic is a square pyramid with a base edge length of 45 feet and a height of 5 yards. What is the volume of the attic in cubic feet? In cubic yards?

Find the volume of each cone. Give your answers both in terms of \( \pi \) and rounded to the nearest tenth.

17. \[
\text{Volume of a cone} = \frac{1}{3} \pi r^2 h \\
= \frac{1}{3} \pi (9)^2 (41) \approx 1097 \\
\]

18. \[
\text{Volume of a cone} = \frac{1}{3} \pi r^2 h \\
= \frac{1}{3} \pi (4)^2 (2) \approx 16.7 \\
\]

19. A cone with base area \( 36\pi \text{ ft}^2 \) and a height equal to twice the radius

Describe the effect of each change on the volume of the given figure.

20. The dimensions are multiplied by \( \frac{1}{3} \).

21. The dimensions are multiplied by 6.

Find the volume of each composite figure. Round to the nearest tenth, if necessary.

22. \[
\text{Volume of a composite figure} = \pi r^2 h \\
= \pi (6)^2 (10) = 360\pi \\
\]

23. \[
\text{Volume of a composite figure} = \frac{1}{3} \pi r^2 h \\
= \frac{1}{3} \pi (3)^2 (5) = 15\pi \\
\]

Find the volume of each right cone with the given dimensions. Give your answers in terms of \( \pi \).

24. Radius 3 in. height 7 in.

25. Diameter 5 m height 2 m

26. Radius 28 ft slant height 53 ft

27. Diameter 24 cm slant height 13 cm
Find the volume of each regular pyramid with the given dimensions. Round to the nearest tenth, if necessary.

<table>
<thead>
<tr>
<th>Number of sides of base</th>
<th>Base edge length</th>
<th>Height</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>28.</td>
<td>3</td>
<td>10 ft</td>
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<tr>
<td>29.</td>
<td>4</td>
<td>15 m</td>
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<tr>
<td>30.</td>
<td>5</td>
<td>9 in.</td>
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<tr>
<td>31.</td>
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<td>8 cm</td>
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<tr>
<td>28.</td>
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<td>15 m</td>
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<tr>
<td>30.</td>
<td>5</td>
<td>9 in.</td>
<td></td>
</tr>
<tr>
<td>31.</td>
<td>6</td>
<td>8 cm</td>
<td></td>
</tr>
</tbody>
</table>

32. Find the height of a rectangular pyramid with length 3 m, width 8 m, and volume 112 m³.

33. Find the base circumference of a cone with height 5 cm and volume 125π cm³.

34. Find the volume of a cone with slant height 10 ft and height 8 ft.

35. Find the volume of a square pyramid with slant height 17 in. and surface area 800 in².

36. Find the surface area of a cone with height 20 yd and volume 1500π yd³.

37. Find the volume of a triangular pyramid with vertices (0, 0, 0), (5, 0, 0), (0, 3, 0), and (0, 0, 7).

38. **ERROR ANALYSIS** Which volume is incorrect? Explain the error.

39. **Critical Thinking** Write a ratio comparing the volume of the prism to the volume of the composite figure. Explain your answer.

40. **Write About It** Explain how you would find the volume of a cone, given the radius and the surface area.

---

41. **Multi-Step Test Prep**

   A juice stand sells smoothies in cone-shaped cups that are 8 in. tall. The regular size has a 4 in. diameter. The jumbo size has an 8 in. diameter.
   
   a. Find the volume of the regular size to the nearest tenth.
   
   b. Find the volume of the jumbo size to the nearest tenth.
   
   c. The regular size costs $1.25. What would be a reasonable price for the jumbo size? Explain your reasoning.
42. Find the volume of the cone.
   \[ \text{A} \quad 432\pi \text{ cm}^3 \quad \text{C} \quad 1296\pi \text{ cm}^3 \]
   \[ \text{B} \quad 720\pi \text{ cm}^3 \quad \text{D} \quad 2160\pi \text{ cm}^3 \]

43. A square pyramid has a slant height of 25 m and a lateral area of 350 m². Which is closest to the volume?
   \[ \text{F} \quad 392 \text{ m}^3 \quad \text{G} \quad 1176 \text{ m}^3 \quad \text{H} \quad 404 \text{ m}^3 \quad \text{J} \quad 1225 \text{ m}^3 \]

44. A cone has a volume of 18\pi \text{ in}^3. Which are possible dimensions of the cone?
   \[ \text{A} \quad \text{Diameter 1 in., height 18 in.} \quad \text{C} \quad \text{Diameter 3 in., height 6 in.} \]
   \[ \text{B} \quad \text{Diameter 6 in., height 6 in.} \quad \text{D} \quad \text{Diameter 6 in., height 3 in.} \]

45. **Gridded Response** Find the height in centimeters of a square pyramid with a volume of 243 cm³ and a base edge length equal to the height.

**CHALLENGE AND EXTEND**

Each cone is inscribed in a regular pyramid with a base edge length of 2 ft and a height of 2 ft. Find the volume of each cone.

46. 47. 48.

49. A regular octahedron has 8 faces that are equilateral triangles. Find the volume of a regular octahedron with a side length of 10 cm.

50. A cylinder has a radius of 5 in. and a height of 3 in. Without calculating the volumes, find the height of a cone with the same base and the same volume as the cylinder. Explain your reasoning.

**SPIRAL REVIEW**

Find the unknown numbers. *(Previous course)*

51. The difference of two numbers is 24. The larger number is 4 less than 3 times the smaller number.

52. Three times the first number plus the second number is 88. The first number times 10 is equal to 4 times the second.

53. The sum of two numbers is 197. The first number is 20 more than \(\frac{1}{2}\) of the second number.

Explain why the triangles are similar, then find each length. *(Lesson 7-3)*

54. \(AB\)

55. \(PQ\)

Find \(AB\) and the coordinates of the midpoint of \(AB\). Round to the nearest tenth, if necessary. *(Lesson 10-3)*

56. \(A(1, 1, 2), B(8, 9, 10)\)

57. \(A(-4, -1, 0), B(5, 1, -4)\)

58. \(A(2, -2, 4), B(-2, 2, -4)\)

59. \(A(-3, -1, 2), B(-1, 5, 5)\)
Functional Relationships in Formulas

You have studied formulas for several solid figures. Here you will see how a change in one dimension affects the measurements of the other dimensions.

Example

A square prism has a volume of 21 cubic units. Write an equation that describes the base edge length \( s \) in terms of the height \( h \). Graph the relationship in a coordinate plane with \( h \) on the horizontal axis and \( s \) on the vertical axis. What happens to the base edge length as the height increases?

First use the volume formula to write an equation.

\[
V = Bh
\]

Substitute 100 for \( V \) and \( s^2 \) for \( B \).

\[
100 = s^2h
\]

Then solve for \( s \) to get an equation for \( s \) in terms of \( h \).

\[
s = \frac{100}{h} \quad \text{Divide both sides by} \ h.
\]

\[
s = \sqrt{\frac{100}{h}} \quad \text{Take the square root of both sides.}
\]

\[
s = \frac{10}{\sqrt{h}} \quad \sqrt{100} = 10
\]

Graph the equation. First make a table of \( h \)- and \( s \)-values. Then plot the points and draw a smooth curve through the points. Notice that the function is not defined for \( h = 0 \).

<table>
<thead>
<tr>
<th>( h )</th>
<th>( s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>3.3</td>
</tr>
<tr>
<td>16</td>
<td>2.5</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
</tr>
</tbody>
</table>

As the height of the prism increases, the base edge length decreases.

Try This

1. A right cone has a radius of 10 units. Write an equation that describes the slant height \( \ell \) in terms of the surface area \( S \). Graph the relationship in a coordinate plane with \( S \) on the horizontal axis and \( \ell \) on the vertical axis. What happens to the slant height as the surface area increases?

2. A cylinder has a height of 5 units. Write an equation that describes the radius \( r \) in terms of the volume \( V \). Graph the relationship in a coordinate plane with \( V \) on the horizontal axis and \( r \) on the vertical axis. What happens to the radius as the volume increases?
A sphere is the locus of points in space that are a fixed distance from a given point called the center of a sphere. A radius of a sphere connects the center of the sphere to any point on the sphere. A hemisphere is half of a sphere. A great circle divides a sphere into two hemispheres.

The figure shows a hemisphere and a cylinder with a cone removed from its interior. The cross sections have the same area at every level, so the volumes are equal by Cavalieri’s Principle. You will prove that the cross sections have equal areas in Exercise 39.

\[
V(\text{hemisphere}) = V(\text{cylinder}) - V(\text{cone})
\]

\[
= \pi r^2 h - \frac{1}{3} \pi r^2 h
\]

\[
= \frac{2}{3} \pi r^2 h
\]

\[
= \frac{2}{3} \pi r^2 (r)
\]

\[
= \frac{2}{3} \pi r^3
\]

The height of the hemisphere is equal to the radius.

The volume of a sphere with radius \( r \) is twice the volume of the hemisphere, or \( V = \frac{4}{3} \pi r^3 \).

**Example 1** Finding Volumes of Spheres

Find each measurement. Give your answer in terms of \( \pi \).

A the volume of the sphere

\[
V = \frac{4}{3} \pi r^3
\]

\[
V = \frac{4}{3} \pi (9)^3
\]

\[
= 972 \pi \text{ cm}^3
\]

Substitute 9 for \( r \).

Simplify.
Find each measurement. Give your answer in terms of $\pi$.

**B**

the diameter of a sphere with volume $972\pi$ in$^3$

1. $972\pi = \frac{4}{3}\pi r^3$  
   Substitute $972\pi$ for $V$.

2. $729 = r^3$  
   Divide both sides by $\frac{4}{3}\pi$.

3. $r = 9$  
   Take the cube root of both sides.

4. $d = 18$ in.  
   $d = 2r$

**C**

the volume of the hemisphere

1. $V = \frac{2}{3}\pi r^3$  
   Volume of a hemisphere

2. $= \frac{2}{3}\pi (4)^3 = \frac{128\pi}{3}$ m$^3$  
   Substitute 4 for $r$.

1. Find the radius of a sphere with volume $2304\pi$ ft$^3$.

**EXAMPLE 2**

**Biology Application**

Giant squid need large eyes to see their prey in low light. The eyeball of a giant squid is approximately a sphere with a diameter of 25 cm, which is bigger than a soccer ball. A human eyeball is approximately a sphere with a diameter of 2.5 cm. How many times as great is the volume of a giant squid eyeball as the volume of a human eyeball?

1. **human eyeball:**
   $V = \frac{4}{3}\pi r^3$
   
   $= \frac{4}{3}\pi (1.25)^3 \approx 8.18$ cm$^3$

2. **giant squid eyeball:**
   $V = \frac{4}{3}\pi r^3$
   
   $= \frac{4}{3}\pi (12.5)^3 \approx 8181.23$ cm$^3$

A giant squid eyeball is about 1000 times as great in volume as a human eyeball.

2. A hummingbird eyeball has a diameter of approximately 0.6 cm. How many times as great is the volume of a human eyeball as the volume of a hummingbird eyeball?

In the figure, the vertex of the pyramid is at the center of the sphere. The height of the pyramid is approximately the radius $r$ of the sphere. Suppose the entire sphere is filled with $n$ pyramids that each have base area $B$ and height $r$.

1. $V(sphere) \approx \frac{1}{3}Br + \frac{1}{3}Br + \ldots + \frac{1}{3}Br$  
   The sphere's volume is close to the sum of the volumes of the pyramids.

2. $\frac{4}{3}\pi r^3 \approx n\left(\frac{1}{3}Br\right)$  
   Divide both sides by $\frac{1}{3}\pi r$.

3. $4\pi r^2 \approx nB$  

If the pyramids fill the sphere, the total area of the bases is approximately equal to the surface area of the sphere $S$, so $4\pi r^2 \approx S$. As the number of pyramids increases, the approximation gets closer to the actual surface area.
The surface area of a sphere with radius \( r \) is \( S = 4\pi r^2 \).

**Example 3** Finding Surface Area of Spheres

Find each measurement. Give your answers in terms of \( \pi \).

**A** the surface area of a sphere with diameter 10 ft

\[
S = 4\pi r^2 \\
S = 4\pi (5)^2 = 200\pi \text{ ft}^2 \\
\text{Substitute 5 for } r.
\]

**B** the volume of a sphere with surface area 144\( \pi \) m\(^2\)

\[
S = 4\pi r^2 \\
144\pi = 4\pi r^2 \\
6 = r \\
V = \frac{4}{3}\pi r^3 \\
\quad = \frac{4}{3}\pi (6)^3 = 288\pi \text{ m}^3 \\
\text{Substitute 6 for } r.
\]

The volume of the sphere is 288\( \pi \) m\(^3\).

**C** the surface area of a sphere with a great circle that has an area of 4\( \pi \) in\(^2\)

\[
\pi r^2 = 4\pi \\
\quad = 4\pi \text{ for } A \text{ in the formula for the area of a circle.} \\
r = 2 \\
S = 4\pi r^2 \\
\quad = 4\pi (2)^2 = 16\pi \text{ in}^2 \\
\text{Substitute 2 for } r \text{ in the surface area formula.}
\]

**Check It Out!**

3. Find the surface area of the sphere.

**Example 4** Exploring Effects of Changing Dimensions

The radius of the sphere is tripled. Describe the effect on the volume.

original dimensions:

\[
V = \frac{4}{3}\pi r^3 \\
\quad = \frac{4}{3}\pi (3)^3 \\
\quad = 36\pi \text{ m}^3
\]

radius tripled:

\[
V = \frac{4}{3}\pi r^3 \\
\quad = \frac{4}{3}\pi (9)^3 \\
\quad = 972\pi \text{ m}^3
\]

Notice that 972\( \pi \) = 27(36\( \pi \)). If the radius is tripled, the volume is multiplied by 27.

**Check It Out!**

4. The radius of the sphere above is divided by 3. Describe the effect on the surface area.
**Example 5** Finding Surface Areas and Volumes of Composite Figures

Find the surface area and volume of the composite figure.
Give your answers in terms of \( \pi \).

**Step 1** Find the surface area of the composite figure.
The surface area of the composite figure is the sum of the surface area of the hemisphere and the lateral area of the cone.

\[
S \text{ (hemisphere)} = \frac{1}{2}(4\pi r^2) = 2\pi(7)^2 = 98\pi \text{ cm}^2
\]

\[
L \text{ (cone)} = \pi r\ell = \pi(7)(25) = 175\pi \text{ cm}^2
\]

The surface area of the composite figure is \( 98\pi + 175\pi = 273\pi \text{ cm}^2 \).

**Step 2** Find the volume of the composite figure.
First find the height of the cone.

\[
h = \sqrt{25^2 - 7^2} \quad \text{Pythagorean Theorem}
\]

\[
h = \sqrt{576} = 24 \text{ cm} \quad \text{Simplify.}
\]

The volume of the composite figure is the sum of the volume of the hemisphere and the volume of the cone.

\[
V \text{ (hemisphere)} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(7)^3 = \frac{686\pi}{3} \text{ cm}^3
\]

\[
V \text{ (cone)} = \frac{1}{3}\pi r^2h = \frac{1}{3}\pi(7)^2(24) = 392\pi \text{ cm}^3
\]

The volume of the composite figure is \( \frac{686\pi}{3} + 392\pi = \frac{1862\pi}{3} \text{ cm}^3 \).

5. Find the surface area and volume of the composite figure.

**Think and Discuss**

1. Explain how to find the surface area of a sphere when you know the area of a great circle.

2. Compare the volume of the sphere with the volume of the composite figure.

3. Get Organized Copy and complete the graphic organizer.
GUDED PRACTICE

1. **Vocabulary** Describe the endpoints of a *radius of a sphere*.

Find each measurement. Give your answers in terms of $\pi$.

2. the volume of the hemisphere

3. the volume of the sphere

4. the radius of a sphere with volume $288\pi$ cm$^3$

5. **Food** Approximately how many times as great is the volume of the grapefruit as the volume of the lime?

Find each measurement. Give your answers in terms of $\pi$.

6. the surface area of the sphere

7. the surface area of the sphere

8. the volume of a sphere with surface area $6724\pi$ ft$^2$

Describe the effect of each change on the given measurement of the figure.

9. surface area

   The dimensions are doubled.

10. volume

   The dimensions are multiplied by $\frac{1}{4}$.

Find the surface area and volume of each composite figure.

11.

12.
PRACTICE AND PROBLEM SOLVING

Find each measurement. Give your answers in terms of $\pi$.

13. the volume of the sphere

14. the volume of the hemisphere

15. the diameter of a sphere with volume $7776\pi$ in$^3$

16. **Jewelry** The size of a cultured pearl is typically indicated by its diameter in mm. How many times as great is the volume of the 9 mm pearl as the volume of the 6 mm pearl?

Find each measurement. Give your answers in terms of $\pi$.

17. the surface area of the sphere

18. the surface area of the sphere

19. the volume of a sphere with surface area $625\pi$ m$^2$

Describe the effect of each change on the given measurement of the figure.

20. surface area
   The dimensions are multiplied by $\frac{1}{5}$.

21. volume
   The dimensions are multiplied by 6.

Find the surface area and volume of each composite figure.

22.

23.

24. Find the radius of a hemisphere with a volume of $144\pi$ cm$^3$.

25. Find the circumference of a sphere with a surface area of $60\pi$ in$^2$.

26. Find the volume of a sphere with a circumference of $36\pi$ ft.

27. Find the surface area and volume of a sphere centered at $(0, 0, 0)$ that passes through the point $(2, 3, 6)$.

28. **Estimation** A bead is formed by drilling a cylindrical hole with a 2 mm diameter through a sphere with an 8 mm diameter. Estimate the surface area and volume of the bead.
Sports  Find the unknown dimensions of the ball for each sport.

<table>
<thead>
<tr>
<th>Sport</th>
<th>Ball</th>
<th>Diameter</th>
<th>Circumference</th>
<th>Surface Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.</td>
<td>Golf</td>
<td>1.68 in.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.</td>
<td>Cricket</td>
<td></td>
<td>9 in.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31.</td>
<td>Tennis</td>
<td>2.5 in.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32.</td>
<td>Petanque</td>
<td>74 mm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Marine Biology  The bathysphere was an early version of a submarine, invented in the 1930s. The inside diameter of the bathysphere was 54 inches, and the steel used to make the sphere was 1.5 inches thick. It had three 8-inch diameter windows. Estimate the volume of steel used to make the bathysphere.

Geography  Earth’s radius is approximately 4000 mi. About two-thirds of Earth’s surface is covered by water. Estimate the land area on Earth.

Astronomy  Use the table for Exercises 35–38.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Diameter (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>3,032</td>
</tr>
<tr>
<td>Venus</td>
<td>7,521</td>
</tr>
<tr>
<td>Earth</td>
<td>7,926</td>
</tr>
<tr>
<td>Mars</td>
<td>4,222</td>
</tr>
<tr>
<td>Jupiter</td>
<td>88,846</td>
</tr>
<tr>
<td>Saturn</td>
<td>74,898</td>
</tr>
<tr>
<td>Uranus</td>
<td>31,763</td>
</tr>
<tr>
<td>Neptune</td>
<td>30,775</td>
</tr>
<tr>
<td>Pluto</td>
<td>1,485</td>
</tr>
</tbody>
</table>

35. How many times as great is the volume of Jupiter as the volume of Earth?
36. The sum of the volumes of Venus and Mars is about equal to the volume of which planet?
37. Which is greater, the sum of the surface areas of Uranus and Neptune or the surface area of Saturn?
38. How many times as great is the surface area of Mercury as the surface area of Pluto?

Critical Thinking  In the figure, the hemisphere and the cylinder both have radius and height $r$. Prove that the shaded cross sections have equal areas.

Write About It  Suppose a sphere and a cube have equal surface areas. Using $r$ for the radius of the sphere and $s$ for the side of a cube, write an equation to show the relationship between $r$ and $s$.

Multi-Step Test Prep  A company sells orange juice in spherical containers that look like oranges. Each container has a surface area of approximately 50.3 in$^2$.

a. What is the volume of the container? Round to the nearest tenth.

b. The company decides to increase the radius of the container by 10%. What is the volume of the new container?
42. A sphere with radius 8 cm is inscribed in a cube. Find the ratio of the volume of the cube to the volume of the sphere.
   \[ \text{A} \: 2: \frac{1}{3} \pi \quad \text{B} \: 2:3 \pi \quad \text{C} \: 1: \frac{4}{3} \pi \quad \text{D} \: 1: \frac{2}{3} \pi \]

43. What is the surface area of a sphere with volume \( 10 \frac{2}{3} \pi \text{ in}^3 \)?
   \[ \text{F} \: 8 \pi \text{ in}^2 \quad \text{G} \: 10 \frac{2}{3} \pi \text{ in}^2 \quad \text{H} \: 16 \pi \text{ in}^2 \quad \text{I} \: 32 \pi \text{ in}^2 \]

44. Which expression represents the volume of the composite figure formed by a hemisphere with radius \( r \) and a cube with side length \( 2r \)?
   \[ \text{A} \: r^3 \left( \frac{2}{3} \pi + 8 \right) \quad \text{C} \: 2r^2(2 \pi + 12) \]
   \[ \text{B} \: \frac{4}{3} \pi r^3 + 2r^3 \quad \text{D} \: \frac{4}{3} \pi r^3 + 8r^3 \]

**CHALLENGE AND EXTEND**

45. **Food** The top of a gumball machine is an 18 in. sphere. The machine holds a maximum of 3300 gumballs, which leaves about 43% of the space in the machine empty. Estimate the diameter of each gumball.

46. The surface area of a sphere can be used to determine its volume.
   a. Solve the surface area formula of a sphere to get an expression for \( r \) in terms of \( S \).
   b. Substitute your result from part a into the volume formula to find the volume \( V \) of a sphere in terms of its surface area \( S \).
   c. Graph the relationship between volume and surface area with \( S \) on the horizontal axis and \( V \) on the vertical axis. What shape is the graph?

Use the diagram of a sphere inscribed in a cylinder for Exercises 47 and 48.

47. What is the relationship between the volume of the sphere and the volume of the cylinder?

48. What is the relationship between the surface area of the sphere and the lateral area of the cylinder?

**SPIRAL REVIEW**

Write an equation that describes the functional relationship for each set of ordered pairs. (*Previous course*)

49. \( \{(0, 1), (1, 2), (-1, 2), (2, 5), (-2, 5)\} \)
50. \( \{(-1, 9), (0, 10), (1, 11), (2, 12), (3, 13)\} \)

Find the shaded area. Round to the nearest tenth, if necessary. (*Lesson 9-3*)

51.

52.

Describe the effect on the volume that results from the given change. (*Lesson 10-6*)

53. The side lengths of a cube are multiplied by \( \frac{3}{4} \).
54. The height and the base area of a prism are multiplied by 5.
Compare Surface Areas and Volumes

In some situations you may need to find the minimum surface area for a given volume. In others you may need to find the maximum volume for a given surface area. Spreadsheet software can help you analyze these problems.

Activity 1

1. Create a spreadsheet to compare surface areas and volumes of rectangular prisms. Create columns for length \(L\), width \(W\), height \(H\), surface area \(SA\), volume \(V\), and ratio of surface area to volume \(SA/V\). In the column for \(SA\), use the formula shown.

2. Create a formula for the \(V\) column and a formula for the \(SA/V\) column.

3. Fill in the measurements \(L = 8\), \(W = 2\), and \(H = 4\) for the first rectangular prism.

4. Choose several values for \(L\), \(W\), and \(H\) to create rectangular prisms that each have the same volume as the first one. Which has the least surface area? Sketch the prism and describe its shape in words. (Is it tall or short, skinny or wide, flat or cubical?) Make a conjecture about what type of shape has the minimum surface area for a given volume.
Try This

1. Repeat Activity 1 for cylinders. Create columns for radius R, height H, surface area SA, volume V, and ratio of surface area to volume SA/V. What shape cylinder has the minimum surface area for a given volume? (Hint: To use \( \pi \) in a formula, input “PI( )” into your spreadsheet.)

2. Investigate packages such as cereal boxes and soda cans. Do the manufacturers appear to be using shapes with the minimum surface areas for their volume? What other factors might influence a company’s choice of packaging?

Activity 2

1. Create a new spreadsheet with the same column headings used in Activity 1. Fill in the measurements \( L = 8, W = 2, \) and \( H = 4 \) for the first rectangular prism. To create a new prism with the same surface area, choose new values for \( L \) and \( W \), and use the formula shown to calculate \( H \).

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>L</td>
<td>W</td>
<td>H</td>
<td>SA</td>
<td>V</td>
<td>SA/V</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>32</td>
<td>112</td>
<td>1.75</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>2</td>
<td>4</td>
<td>112</td>
<td>64</td>
<td>1.75</td>
</tr>
</tbody>
</table>

2. Choose several more values for \( L \) and \( W \), and calculate \( H \) so that \( SA = 112 \). Examine the \( V \) and \( SA/V \) columns. Which prism has the greatest volume? Sketch the prism and describe it in words. Make a conjecture about what type of shape has the maximum volume for a given surface area.

Try This

3. Repeat Activity 2 for cylinders. Create columns for radius \( R \), height \( H \), surface area \( SA \), volume \( V \), and the ratio of surface area to volume \( SA/V \). What shape cylinder has the maximum volume for a given surface area?

4. Solve the formula \( SA = 2LW + 2LH + 2WH \) for \( H \). Use your result to explain the formula that was used to find \( H \) in Activity 2.

5. If a rectangular prism, a pyramid, a cylinder, a cone, and a sphere all had the same volume, which do you think would have the least surface area? Which would have the greatest surface area? Explain.

6. Use a spreadsheet to analyze what happens to the ratio of surface area to volume of a rectangular prism when the dimensions are doubled. Explain how you set up the spreadsheet and describe your results.
Surface Area and Volume

Juice for Fun  You are in charge of designing containers for a new brand of juice. Your company wants you to compare several different container shapes. The container must be able to hold a 6-inch straw so that exactly 1 inch remains outside the container when the straw is inserted as far as possible.

1. One possible container is a cylinder with a base diameter of 4 in., as shown. How much material is needed to make this container? Round to the nearest tenth.

2. Estimate the volume of juice in ounces that the cylinder will hold. Round to the nearest tenth.  
   \((\text{Hint: } 1 \text{ in}^3 \approx 0.55 \text{ oz})\)

3. Another option is a square prism with a 3 in. by 3 in. base, as shown. How much material is needed to make this container?

4. Estimate the volume of juice in ounces that the prism will hold.  
   \((\text{Hint: } 1 \text{ in}^3 \approx 0.55 \text{ oz})\)

5. Which container would you recommend to your company? Justify your answer.
Quiz for Lessons 10-4 Through 10-8

10-4 Surface Area of Prisms and Cylinders
Find the surface area of each figure. Round to the nearest tenth, if necessary.

1. 

2. 

3. 

4. The dimensions of a 12 mm by 8 mm by 24 mm right rectangular prism are multiplied by \( \frac{3}{4} \). Describe the effect on the surface area.

10-5 Surface Area of Pyramids and Cones
Find the surface area of each figure. Round to the nearest tenth, if necessary.

5. a regular pentagonal pyramid with base edge length 18 yd and slant height 20 yd

6. a right cone with diameter 30 in. and height 8 in.

7. the composite figure formed by two cones

10-6 Volume of Prisms and Cylinders
Find the volume of each figure. Round to the nearest tenth, if necessary.

8. a regular hexagonal prism with base area 23 in\(^2\) and height 9 in.

9. a cylinder with radius 8 yd and height 14 yd

10. A brick patio measures 10 ft by 12 ft by 4 in. Find the volume of the bricks. If the density of brick is 130 pounds per cubic foot, what is the weight of the patio in pounds?

11. The dimensions of a cylinder with diameter 2 ft and height 1 ft are doubled. Describe the effect on the volume.

10-7 Volume of Pyramids and Cones
Find the volume of each figure. Round to the nearest tenth, if necessary.

12. 

13. 

14. 

10-8 Spheres
Find the surface area and volume of each figure.

15. a sphere with diameter 20 in. 

16. a hemisphere with radius 12 in.

17. A baseball has a diameter of approximately 3 in., and a softball has a diameter of approximately 5 in. About how many times as great is the volume of a softball as the volume of a baseball?
Spherical Geometry

**Objective**
Understand spherical geometry as an example of non-Euclidean geometry.

**Vocabulary**
- non-Euclidean geometry
- spherical geometry

Euclidean geometry is based on figures in a plane. **Non-Euclidean geometry** is based on figures in a curved surface. In a non-Euclidean geometry system, the Parallel Postulate is not true. One type of non-Euclidean geometry is **spherical geometry**, which is the study of figures on the surface of a sphere.

A line in Euclidean geometry is the shortest path between two points. On a sphere, the shortest path between two points is along a great circle, so “lines” in spherical geometry are defined as great circles. In spherical geometry, there are no parallel lines. Any two lines intersect at two points.

**Spherical Geometry Parallel Postulate**

Through a point not on a line, there is no line parallel to a given line.

**Example 1**

**Classifying Figures in Spherical Geometry**

Name a line, a segment, and a triangle on the sphere.

- \( \overrightarrow{AC} \) is a line.
- \( \overline{AC} \) is a segment.
- \( \triangle ACD \) is a triangle.

1. Name another line, segment, and triangle on the sphere above.

In Example 1, the lines \( \overrightarrow{AC} \) and \( \overrightarrow{AD} \) are both perpendicular to \( \overrightarrow{CD} \). This means that \( \triangle ACD \) has two right angles. So the sum of its angle measures must be greater than 180°.

Imagine cutting an orange in half and then cutting each half in quarters using two perpendicular cuts. Each of the resulting triangles has three right angles.

**Spherical Triangle Sum Theorem**

The sum of the angle measures of a spherical triangle is greater than 180°.
Example 2: Classifying Spherical Triangles

Classify each spherical triangle by its angle measures and by its side lengths.

A \( \triangle ABC \)
\( \triangle ABC \) is an obtuse scalene triangle.

B \( \triangle NPQ \) on Earth has vertex \( N \) at the North Pole and vertices \( P \) and \( Q \) on the equator.
\( PQ \) is equal to \( \frac{1}{3} \) the circumference of Earth.
\( NP \) and \( NQ \) are both equal to \( \frac{1}{4} \) the circumference of Earth.
The equator is perpendicular to both of the other two sides of the triangle. Thus \( \triangle NPQ \) is an isosceles right triangle.

2. Classify \( \triangle VWX \) by its angle measures and by its side lengths.

The area of a spherical triangle is part of the surface area of the sphere. For the piece of orange on page 726, the area is \( \frac{1}{8} \) of the surface area of the orange, or \( \frac{1}{8} (4\pi r^2) = \frac{\pi r^2}{2} \). If you know the radius of a sphere and the measure of each angle, you can find the area of the triangle.

Area of a Spherical Triangle

The area of spherical \( \triangle ABC \) on a sphere with radius \( r \) is
\[
A = \frac{\pi r^2}{180^\circ} (m\angle A + m\angle B + m\angle C - 180^\circ).
\]

Example 3: Finding the Area of Spherical Triangles

Find the area of each spherical triangle. Round to the nearest tenth, if necessary.

A \( \triangle ABC \)
\[
A = \frac{\pi r^2}{180^\circ} (m\angle A + m\angle B + m\angle C - 180^\circ)
\]
\[
A = \frac{\pi (14)^2}{180^\circ} (100 + 106 + 114 - 180) \approx 152.4 \text{ cm}^2
\]

B \( \triangle DEF \) on Earth's surface with \( m\angle D = 75^\circ \), \( m\angle E = 80^\circ \), and \( m\angle F = 30^\circ \). (Hint: average radius of Earth = 3959 miles)
\[
A = \frac{\pi r^2}{180^\circ} (m\angle D + m\angle E + m\angle F - 180^\circ)
\]
\[
= \frac{\pi (3959)^2}{180^\circ} (75 + 80 + 30 - 180) \approx 1,367,786.7 \text{ mi}^2
\]

3. Find the area of \( \triangle KLM \) on a sphere with diameter 20 ft, where \( m\angle K = 90^\circ \), \( m\angle L = 90^\circ \), and \( m\angle M = 30^\circ \). Round to the nearest tenth.
Use the figure for Exercises 1–3.
1. Name all lines on the sphere.
2. Name three segments on the sphere.
3. Name a triangle on the sphere.

Determine whether each figure is a line in spherical geometry.
4. \( m \)
5. \( n \)
6. \( p \)

Classify each spherical triangle by its angle measures and by its side lengths.
7. 
8. 
9. 
10. 

Find the area of each spherical triangle.
11. 
12. 

13. \( \triangle XYZ \) on the Moon’s surface with \( \angle A = 35^\circ \), \( \angle B = 48^\circ \), and \( \angle C = 100^\circ \) (Hint: average radius of the Moon \( \approx 1079 \) miles)

14. \( \triangle RST \) on a scale model of Earth with radius 6 m, \( \angle R = 80^\circ \), \( \angle S = 130^\circ \), and \( \angle T = 150^\circ \)
17. \( \triangle ABC \) is an acute triangle.
   a. Write an inequality for the sum of the angle measures of \( \triangle ABC \), based on the fact that \( \triangle ABC \) is acute.
   b. Use your result from part a to write an inequality for the area of \( \triangle ABC \).
   c. Use your result from part b to compare the area of an acute spherical triangle to the total surface area of the sphere.

18. Draw a quadrilateral on a sphere. Include one diagonal in your drawing. Use the sum of the angle measures of the quadrilateral to write an inequality.

**Geography** Compare each length to the length of a great circle on Earth.

19. the distance between the North Pole and the South Pole
20. the distance between the North Pole and any point on the equator

21. **Geography** If the area of a triangle on Earth’s surface is 100,000 \( \text{mi}^2 \), what is the sum of its angle measures? (Hint: average radius of Earth \( \approx 3959 \text{ miles} \))

22. **Sports** Describe the curves on the basketball that are lines in spherical geometry.

23. **Navigation** Pilots navigating long distances often travel along the lines of spherical geometry. Using a globe and string, determine the shortest route for a plane traveling from Washington, D.C., to London, England. What do you notice?

24. **Write About It** Can a spherical triangle be right and obtuse at the same time? Explain.

25. **Write About It** A 2-gon is a polygon with two edges. Draw two lines on a sphere. How many 2-gons are formed? What can you say about the positions of the vertices of the 2-gons on the sphere?

26. **Challenge** Another type of non-Euclidean geometry, called hyperbolic geometry, is defined on a surface that is curved like the bell of a trumpet. What do you think is true about the sum of the angle measures of the triangle shown at right? Compare the sum of the angle measures of a triangle in Euclidean, spherical, and hyperbolic geometry.
**Vocabulary**

- altitude ....................... 680
- altitude of a cone ............ 690
- altitude of a pyramid .......... 689
- axis of a cone .................. 690
- axis of a cylinder .............. 681
- center of a sphere .......... 714
- cone .......................... 654
- cross section ................... 656
- cube ........................... 654
- cylinder ......................... 654
- edge ............................ 654
- face ............................. 654
- great circle ...................... 714
- hemisphere ...................... 714
- horizon ......................... 662
- isometric drawing .............. 662
- lateral edge ...................... 680
- lateral face ....................... 680
- lateral surface .................. 681
- net .................................. 655
- oblique cone ..................... 690
- oblique cylinder .......... 681
- oblique prism .................... 680
- orthographic drawing ............ 661
- perspective drawing .............. 662
- polyhedron ...................... 670
- prism ............................ 654
- pyramid ......................... 654
- radius of a sphere .......... 714
- regular pyramid ............... 689
- right cone ....................... 690
- right cylinder ................... 681
- right prism ...................... 680
- slant height of a regular pyramid ... 689
- slant height of a right cone ........ 690
- space ................................... 671
- sphere ........................... 714
- surface area ...................... 680
- vanishing point ................. 662
- vertex ............................ 654
- vertex of a cone ............... 690
- vertex of a pyramid .............. 689
- volume ............................ 697

Complete the sentences below with vocabulary words from the list above.

1. A(n) ____ has at least one nonrectangular lateral face.
2. A name given to the intersection of a three-dimensional figure and a plane is ____.

**EXERCISES**

### EXERCISES

#### 10-1 Solid Geometry (pp. 654–660)

**EXAMPLES**

- Classify the figure. Name the vertices, edges, and bases.
  - pentagonal prism
  - vertices: A, B, C, D, E, F, G, H, J, K
  - edges: AB, BC, CD, DE, AE, FG, GH, HJ, JK, KP, AF, ED, DJ, CH, BG
  - bases: ABCDE, FGHJK

- Describe the three-dimensional figure that can be made from the given net.
  - The net forms a rectangular prism.

**EXERCISES**

Classify each figure. Name the vertices, edges, and bases.

3. [Diagram of a cone]
4. [Diagram of a pyramid]

Describe the three-dimensional figure that can be made from the given net.

5. [Diagram of a net for a rectangular prism]
6. [Diagram of a net for a tetrahedron]
10-2 Representations of Three-Dimensional Figures (pp. 661–668)

**Examples**
- Draw all six orthographic views of the given object. Assume there are no hidden cubes.
  - Top: ![Top View](image1)
  - Bottom: ![Bottom View](image2)
  - Front: ![Front View](image3)
  - Back: ![Back View](image4)
  - Left: ![Left View](image5)
  - Right: ![Right View](image6)
- Draw an isometric view of the given object. Assume there are no hidden cubes.

**Exercises**
- Use the figure made of unit cubes for Exercises 7–10. Assume there are no hidden cubes.
  1. Draw all six orthographic views.
  2. Draw an isometric view.
  3. Draw the object in one-point perspective.
  4. Draw the object in two-point perspective.
- Determine whether each drawing represents the given object. Assume there are no hidden cubes.

10-3 Formulas in Three Dimensions (pp. 670–677)

**Examples**
- Find the number of vertices, edges, and faces of the given polyhedron. Use your results to verify Euler's formula.
  \[ V = 12, \ E = 18, \ F = 8 \]
  \[ 12 - 18 + 8 = 2 \]
- Find the distance between the points \((6, 3, 4)\) and \((2, 7, 9)\). Find the midpoint of the segment with the given endpoints. Round to the nearest tenth, if necessary.
  \[ d = \sqrt{(2 - 6)^2 + (7 - 3)^2 + (9 - 4)^2} \]
  \[ d = \sqrt{57} \approx 7.5 \]
  \[ M(\frac{6 + 2}{2}, \frac{3 + 7}{2}, \frac{4 + 9}{2}) \]
  \[ M(4, 5, 6.5) \]

**Exercises**
- Find the number of vertices, edges, and faces of each polyhedron. Use your results to verify Euler's formula.
  13. ![Shape 13](image7)
  14. ![Shape 14](image8)
- Find the distance between the given points. Find the midpoint of the segment with the given endpoints. Round to the nearest tenth, if necessary.
  15. \((2, 6, 4)\) and \((7, 1, 1)\)
  16. \((0, 3, 0)\) and \((5, 7, 8)\)
  17. \((7, 2, 6)\) and \((9, 1, 5)\)
  18. \((6, 2, 8)\) and \((2, 7, 4)\)
10-4 Surface Area of Prisms and Cylinders (pp. 680–687)

**EXAMPLES**

Find the lateral area and surface area of each right prism or cylinder.

- A right prism with a base that is a rectangle 7 in. by 10 in. and a height of 7 in.
  
  \[
  L = Ph = 28(10) = 280 \text{ in}^2 \\
  S = Ph + 2B = 280 + 2(49) = 378 \text{ in}^2
  \]

- A cylinder with radius 8 m and height 12 m.
  
  \[
  L = 2\pi rh = 2\pi(8)(12) = 192\pi \approx 603.2 \text{ m}^2 \\
  S = L + 2B = 192\pi + 2\pi(8)^2 = 320\pi \approx 1005.3 \text{ m}^2
  \]

**EXERCISES**

Find the lateral area and surface area of each right prism or cylinder. Round to the nearest tenth, if necessary.

- Find the lateral area and surface area of each right prism or cylinder. Round to the nearest tenth, if necessary.
  
  \[
  \begin{align*}
  19. & \quad \text{a cube with side length 5 ft} \\
  20. & \quad \text{an equilateral triangular prism with height 7 m and base edge lengths 6 m} \\
  21. & \quad \text{a regular pentagonal prism with height 8 cm and base edge length 4 cm}
  \end{align*}
  \]

10-5 Surface Area of Pyramids and Cones (pp. 689–696)

**EXAMPLES**

Find the lateral area and surface area of each right pyramid or cone.

- A right triangle with sides 15 m, 16 m, and 15 m.
  
  \[
  L = \frac{1}{2}P\ell = \frac{1}{2}(48)(20) = 480 \text{ in}^2 \\
  S = L + B = 480 + \frac{1}{2}(4\sqrt{3})(48) \approx 646.3 \text{ in}^2
  \]

- A regular hexagonal pyramid with base edge length 8 in. and slant height 20 in.
  
  \[
  L = \frac{1}{2}P\ell = \frac{1}{2}(48)(20) = 480 \text{ in}^2 \\
  S = L + B = 480 + \frac{1}{2}(4\sqrt{3})(48) \approx 646.3 \text{ in}^2
  \]

**EXERCISES**

Find the lateral area and surface area of each right pyramid or cone.

- Find the lateral area and surface area of each right pyramid or cone.
  
  \[
  \begin{align*}
  23. & \quad \text{a square pyramid with side length 15 ft and slant height 21 ft} \\
  24. & \quad \text{a cone with radius 7 m and height 24 m} \\
  25. & \quad \text{a cone with diameter 20 in. and slant height 15 in.}
  \end{align*}
  \]

10-6 Volume of Prisms and Cylinders (pp. 697–704)

**EXAMPLES**

Find the volume of the prism.

- A rectangular prism with dimensions 8 cm, 12 cm, and 12 cm.
  
  \[
  V = Bh = \left(\frac{1}{2}aP\right)h \\
  = \frac{1}{2}(4\sqrt{3})(48)(12) \\
  = 1152\sqrt{3} \approx 1995.3 \text{ cm}^3
  \]

**EXERCISES**

Find the volume of each prism.

- Find the volume of each prism.
  
  \[
  \begin{align*}
  28. & \quad \text{a rectangular prism with dimensions 9 ft, 10 ft, and 12 ft} \\
  29. & \quad \text{a rectangular prism with dimensions 8 cm, 15 cm, and 12 cm}
  \end{align*}
  \]
10-7 Volume of Pyramids and Cones (pp. 705–712)

**EXAMPLES**

- Find the volume of the pyramid.
  \[ V = \frac{1}{3}Bh = \frac{1}{3}(8 \cdot 3)(14) \]
  \[ = 112 \text{ in}^3 \]

- Find the volume of the cone.
  \[ V = \frac{1}{3}\pi r^2h = \frac{1}{3}\pi(9)^2(16) \]
  \[ = 432\pi \text{ ft}^3 \approx 1357.2 \text{ ft}^3 \]

**EXERCISES**

Find the volume of each pyramid or cone.

32. a hexagonal pyramid with base area 42 m² and height 8 m
33. an equilateral triangular pyramid with base edge 3 cm and height 8 cm
34. a cone with diameter 12 cm and height 10 cm
35. a cone with base area $16\pi$ ft² and height 9 ft

Find the volume of each composite figure.

36. 
37. 

Find each measurement. Give your answers in terms of $\pi$.

38. the volume of a sphere with surface area $100\pi$ m²
39. the surface area of a sphere with volume $288\pi$ in³
40. the diameter of a sphere with surface area $256\pi$ ft²

Find the surface area and volume of each composite figure.

41. 
42. 

10-8 Spheres (pp. 714–721)

**EXAMPLE**

- Find the volume and surface area of the sphere. Give your answers in terms of $\pi$.
  \[ V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(9)^3 = 972\pi \text{ m}^3 \]
  \[ S = 4\pi r^2 = 4\pi(9)^2 = 324\pi \text{ m}^2 \]

**EXERCISES**

Find each measurement. Give your answers in terms of $\pi$.

38. the volume of a sphere with surface area $100\pi$ m²
39. the surface area of a sphere with volume $288\pi$ in³
40. the diameter of a sphere with surface area $256\pi$ ft²

Find the surface area and volume of each composite figure.

41. 
42. 

Study Guide: Review
Use the diagram for Items 1–3.
1. Classify the figure. Name the vertices, edges, and bases.
2. Describe a cross section made by a plane parallel to the base.
3. Find the number of vertices, edges, and faces of the polyhedron. Use your results to verify Euler’s formula.

Use the figure made of unit cubes for Items 4–6. Assume there are no hidden cubes.
4. Draw all six orthographic views.
5. Draw an isometric view.
6. Draw the object in one-point perspective.

Find the distance between the given points. Find the midpoint of the segment with the given endpoints. Round to the nearest tenth, if necessary.
7. \((0, 0, 0)\) and \((5, 5, 5)\)
8. \((6, 0, 9)\) and \((7, 1, 4)\)
9. \((-1, 4, 3)\) and \((2, -5, 7)\)

Find the surface area of each figure. Round to the nearest tenth, if necessary.
10. \[
\begin{array}{c}
\text{front view} \\
\text{top view} \\
\text{side view}
\end{array}
\]
11. \[
\begin{array}{c}
\text{top view} \\
\text{side view}
\end{array}
\]
12. \[
\begin{array}{c}
\text{front view} \\
\text{side view}
\end{array}
\]
13. \[
\begin{array}{c}
\text{frustum of a cone}
\end{array}
\]
14. \[
\begin{array}{c}
\text{sphere}
\end{array}
\]
15. \[
\begin{array}{c}
\text{rectangular prism}
\end{array}
\]

Find the volume of each figure. Round to the nearest tenth, if necessary.
16. \[
\begin{array}{c}
\text{cuboid}
\end{array}
\]
17. \[
\begin{array}{c}
\text{right circular cone}
\end{array}
\]
18. \[
\begin{array}{c}
\text{frustum of a cone}
\end{array}
\]
19. \[
\begin{array}{c}
\text{cylinder}
\end{array}
\]
20. \[
\begin{array}{c}
\text{right circular cone}
\end{array}
\]
21. \[
\begin{array}{c}
\text{sphere}
\end{array}
\]

22. Earth’s diameter is approximately 7930 miles. The Moon’s diameter is approximately 2160 miles. About how many times as great is the volume of Earth as the volume of the Moon?
FOCUS ON SAT MATHEMATICS SUBJECT TEST

SAT Mathematics Subject Test results include scaled scores and percentiles. Your scaled score is a number from 200 to 800, calculated using a formula that varies from year to year. The percentile indicates the percentage of people who took the same test and scored lower than you did.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. A line intersects a cube at two points, $A$ and $B$. If each edge of the cube is 4 cm, what is the greatest possible distance between $A$ and $B$?
   (A) $2\sqrt{3}$ cm
   (B) 4 cm
   (C) $4\sqrt{2}$ cm
   (D) $4\sqrt{3}$ cm
   (E) $16\sqrt{3}$ cm

2. The lateral area of a right cylinder is 3 times the area of its base. What is the height $h$ of the cylinder in terms of its radius $r$?
   (A) $\frac{1}{2}r$
   (B) $\frac{2}{3}r$
   (C) $\frac{3}{2}r$
   (D) $3r$
   (E) $3r^2$

3. What is the lateral area of a right cone with radius 6 ft and height 8 ft?
   (A) $30\pi$ ft$^2$
   (B) $48\pi$ ft$^2$
   (C) $60\pi$ ft$^2$
   (D) $180\pi$ ft$^2$
   (E) $360\pi$ ft$^2$

4. If triangle $ABC$ is rotated about the $x$-axis, what is the volume of the resulting cone?

(Click to view diagram)

   (A) $100\pi$ cubic units
   (B) $144\pi$ cubic units
   (C) $240\pi$ cubic units
   (D) $300\pi$ cubic units
   (E) $720\pi$ cubic units

5. An oxygen tank is the shape of a cylinder with a hemisphere at each end. If the radius of the tank is 5 inches and the overall length is 32 inches, what is the volume of the tank?
   (A) $\frac{500}{3}\pi$ in$^3$
   (B) $\frac{2275}{12}\pi$ in$^3$
   (C) $\frac{1900}{3}\pi$ in$^3$
   (D) $\frac{2150}{3}\pi$ in$^3$
   (E) $\frac{2900}{3}\pi$ in$^3$
Any Question Type: Measure to Solve Problems

On some tests, you may have to measure a figure in order to answer a question. Pay close attention to the units of measure asked for in the question. Some questions ask you to measure to the nearest centimeter, and some ask you to measure to the nearest inch.

**Example 1**

**Multiple Choice:** The net of a square pyramid is shown below. Use a ruler to measure the dimensions of the pyramid to the nearest centimeter.

Which of the following best represents the total surface area of the square pyramid?

- A) 9 square centimeters
- B) 21 square centimeters
- C) 33 square centimeters
- D) 36 square centimeters

**Solution:**

Use a centimeter ruler to measure one side of the square base. The measurement to the nearest centimeter is 3 cm. The base is a square, so all four side lengths are 3 cm.

Measure the altitude of a triangular face, which is the slant height of the pyramid. The altitude is 2 cm. Label the drawing with the measurements.

To find the total surface area of the pyramid, find the base area and the lateral area.

The base of the pyramid is a square. The base area of the pyramid is \( A = s^2 = (3)^2 = 9 \) cm².

The area of one triangular face is \( A = \frac{1}{2}bh = \frac{1}{2}(3)(2) = 3 \) cm².

The pyramid has 4 faces, so the lateral area is \( 4(3) = 12 \) cm².

The total surface area is \( 9 + 12 = 21 \) cm². The correct answer choice is B.
Read each test item and answer the questions that follow.

**Item A**
The net of a cube is shown below. Use a ruler to measure the dimensions of the cube to the nearest \( \frac{1}{4} \) inch.

Which best represents the volume of the cube to the nearest cubic inch?

A. 1 cubic inch  
B. 2 cubic inches  
C. 5 cubic inches  
D. 9 cubic inches

1. Measure one edge of the net for the cube. What is the length to the nearest \( \frac{1}{4} \) inch?
2. How would you use the measurement to find the volume of the cube?

**Item B**
The net of a cylinder is shown below. Use a ruler to measure the dimensions of the cylinder to the nearest tenth of a centimeter.

Which best represents the total surface area of the cylinder to the nearest square centimeter?

F. 6 square centimeters  
G. 16 square centimeters  
H. 19 square centimeters  
I. 42 square centimeters

3. Which part of the net do you need to measure in order to find the height of the cylinder? Find the height of the cylinder to the nearest tenth of a centimeter.
4. What other measurement(s) do you need in order to find the surface area of the cylinder? Find the measurement(s) to the nearest tenth of a centimeter.
5. How would you use the measurements to find the surface area of the cylinder?
CUMULATIVE ASSESSMENT, CHAPTERS 1–10

Multiple Choice

1. If a point \((x, y)\) is chosen at random in the coordinate plane such that \(-1 \leq x \leq 1\) and \(-5 \leq y \leq 3\), what is the probability that \(x \geq 0\) and \(y \geq 0\)?
   - \(\text{A} \ 0.1875\)
   - \(\text{B} \ 0.25\)
   - \(\text{C} \ 0.375\)
   - \(\text{D} \ 0.8125\)

2. \(\triangle ABC \sim \triangle DEF\), and \(\triangle DEF \sim \triangle GHI\). If the similarity ratio of \(\triangle ABC\) to \(\triangle DEF\) is \(\sqrt{2}\) and the similarity ratio of \(\triangle DEF\) to \(\triangle GHI\) is \(\frac{3}{4}\), what is the similarity ratio of \(\triangle ABC\) to \(\triangle GHI\)?
   - \(\text{A} \ \frac{1}{4}\)
   - \(\text{B} \ \frac{2}{3}\)
   - \(\text{C} \ \frac{3}{8}\)
   - \(\text{D} \ \frac{3}{2}\)

3. Which expression represents the number of faces of a prism with bases that are \(n\)-gons?
   - \(\text{A} \ n + 1\)
   - \(\text{B} \ n + 2\)
   - \(\text{C} \ 2n\)
   - \(\text{D} \ 3n\)

4. Parallelogram \(ABCD\) has a diagonal \(\overline{AC}\) with endpoints \(A(-1, 3)\) and \(C(-3, -3)\). If \(B\) has coordinates \((x, y)\), which of the following represents the coordinates for \(D\)?
   - \(\text{A} \ D(-3x, -y)\)
   - \(\text{B} \ D(-x, -y)\)
   - \(\text{C} \ D(-x - 4, -y)\)
   - \(\text{D} \ D(x - 2, y)\)

5. Right \(\triangle ABC\) with legs \(AB = 9\) millimeters and \(BC = 12\) millimeters is the base of a right prism that has a surface area of 450 square millimeters. What is the height of the prism?
   - \(\text{A} \ 4.75\) millimeters
   - \(\text{B} \ 6\) millimeters
   - \(\text{C} \ 9.5\) millimeters
   - \(\text{D} \ 11\) millimeters

6. The radius of a sphere is doubled. What happens to the ratio of the volume of the sphere to the surface area of the sphere?
   - \(\text{A} \ \text{It remains the same.}\)
   - \(\text{B} \ \text{It is doubled.}\)
   - \(\text{C} \ \text{It is increased by a factor of 4.}\)
   - \(\text{D} \ \text{It is increased by a factor of 8.}\)

7. \(\overline{AB}\) has endpoints \(A(x, y, z)\) and \(B(-2, 6, 13)\) and midpoint \(M(2, -6, 3)\). What are the coordinates of \(A\)?
   - \(\text{A} \ (0, -18, -7)\)
   - \(\text{B} \ (A(0, 0, 8)\)
   - \(\text{C} \ (2, -6, 19)\)
   - \(\text{D} \ (6, -18, 23)\)

8. If \(\overline{DE}\) bisects \(\angle CEF\), which of the following additional statements would allow you to conclude that \(\triangle DEF \cong \triangle ABC\)?
   - \(\text{A} \ \angle DEF \cong \angle BAC\)
   - \(\text{B} \ \angle DEF \cong \angle CDE\)
   - \(\text{C} \ \overline{EF} \cong \overline{CD}\)
   - \(\text{D} \ \overline{EF} \cong \overline{EC}\)

9. To the nearest tenth of a cubic centimeter, what is the volume of a right regular octagonal prism with base edge length 4 centimeters and height 7 centimeters?
   - \(\text{A} \ 180.3\) cubic centimeters
   - \(\text{B} \ 224.0\) cubic centimeters
   - \(\text{C} \ 270.4\) cubic centimeters
   - \(\text{D} \ 540.8\) cubic centimeters

10. Which of the following must be true about a conditional statement?
    - \(\text{A} \ \text{If the inverse is false, then the converse is false.}\)
    - \(\text{B} \ \text{If the conditional is true, then the contrapositive is false.}\)
    - \(\text{C} \ \text{If the conditional is true, then the converse is false.}\)
    - \(\text{D} \ \text{If the hypothesis of the conditional is true, then the conditional is true.}\)
11. A right cylinder has a height of 10 inches. The area of the base is 63.6 square inches. To the nearest tenth of a square inch, what is the lateral area for this cylinder?

A. 53.6 square inches  
B. 282.7 square inches  
C. 409.9 square inches  
D. 634.6 square inches

12. The volume of the smaller sphere is 288 cubic centimeters. Find the volume of the larger sphere.

F. 864 cubic centimeters  
G. 2,592 cubic centimeters  
H. 7,776 cubic centimeters  
J. 23,328 cubic centimeters

**Gridded Response**

13. $\mathbf{u} = (3, -7)$, and $\mathbf{v} = (-6, 5)$. What is the magnitude of the resultant vector to the nearest tenth of $\mathbf{u}$ and $\mathbf{v}$?

14. If a polyhedron has 12 vertices and 8 faces, how many edges does the polyhedron have?

15. If Y is the circumcenter of $\triangle PQR$, what is the value of $x$?

16. How many cubes with edge length 3 centimeters will fit in a box that is a rectangular prism with length 12 centimeters, width 15 centimeters, and height 24 centimeters?

**Short Response**

17. The area of trapezoid $GHIJ$ is 103.5 square centimeters. Find each of the following. Round answers to the nearest tenth. Show your work or explain in words how you found your answers.

![Trapezoid Diagram]

a. the height of trapezoid $GHIJ$  
b. $m \angle J$

18. The figure shows the top view of a stack of cubes. The number on each cube represents the number of stacked cubes. Draw the bottom, back, and right views of the object.

![Cube Stack Diagram]

19. $\triangle ABC$ has vertices $A(1, -2), B(-2, -3),$ and $C(-2, 2)$.

a. Graph $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation with a scale factor of $\frac{3}{2}$.

b. Show that $\overline{AB} \parallel \overline{A'B'}, \overline{BC} \parallel \overline{B'C'},$ and $\overline{CA} \parallel \overline{C'A'}$. Use slope to justify your answer.

**Extended Response**

20. A right cone has a lateral area of $30\pi$ square inches and a slant height of 6 inches.

a. Find the height of the cone. Show your work or explain in words how you determined your answer. Round your answer to the nearest tenth.

b. Find the volume of this cone. Round your answer to the nearest tenth.

c. Given a right cone with a lateral area of $L$ and a slant height of $l$, find an equation for the volume in terms of $L$ and $l$. Show your work.
The Mellon Arena

When Pittsburgh’s Mellon Arena opened in 1961, it was the world’s first auditorium with a retractable roof.

Choose one or more strategies to solve each problem.

1. The Mellon Arena appears to be a perfect circle. However it actually consists of two semi-circles that are connected by a narrow rectangle, as shown in the figure. Approximately how many acres of land does the arena cover? (*Hint: 1 acre = 43,560 ft²*)

For 2 and 3, use the table.

2. For hockey games, the arena’s standard rectangular floor is used. The ratio of the floor’s length to its width is 40:17. What are the dimensions of the standard arena floor?

3. For special events, some of the seating can be removed to create an expanded rectangular floor. In this case, the length is 130 ft greater than the width. What are the dimensions of the expanded arena floor?

4. The arena’s roof is a stainless steel dome. It consists of eight congruent wedge-shaped sections. When the roof is retracted, six of the sections rotate and come to rest under the two sections that remain fixed. Suppose you choose a seat in the arena at random. When the roof is retracted, what is the probability that you are sitting under one of the fixed sections? What is the probability that you are sitting under the open sky?

<table>
<thead>
<tr>
<th>Mellon Arena Floor</th>
<th>Perimeter (ft)</th>
<th>Area (ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>570</td>
<td>17,000</td>
</tr>
<tr>
<td>Expanded</td>
<td>740</td>
<td>30,000</td>
</tr>
</tbody>
</table>
Chances are good that you have a souvenir of Philadelphia in your pocket. Since 1792, the U.S. mint has had a facility in Philadelphia, and over the years it has produced trillions of coins. In 2004 alone, the Philadelphia mint turned out approximately 3 billion pennies.

Choose one or more strategies to solve each problem. For 1–4, use the table.

1. Coins are stamped out of a rectangular metal strip that is 13 in. wide by 1,500 ft long. Given that the diameter of a quarter is just under an inch (0.955 in.), what is the minimum number of strips that would be needed to stamp out 700,000 quarters?

2. A penny contains a small amount of copper, but most of the metal in a penny is zinc. The volume of copper in a penny is about $11 \text{ mm}^3$. What percentage of the penny is copper?

3. Nickels are made from a metal that is a mixture of nickel and copper. About how many nickels can be made if a 1 m$^3$ block of this metal is melted down?

4. Rolls of 50 dimes are packaged in clear plastic sleeves. How much plastic is needed to enclose one roll of dimes?