1. A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested.

(a) Write down the number of defective lamps expected in the sample. [1 mark]

(b) Find the probability that there are exactly five defective lamps in the sample. [3 marks]

(c) Find the probability that there are fewer than five defective lamps in the sample. [3 marks]

2. Evan likes to play two games of chance, A and B.

For game A, the probability that Evan wins is 0.9. He plays game A seven times.

(a) Find the probability that he wins exactly four games. [2 marks]

For game B, the probability that Evan wins is $p$. He plays game B seven times.

(b) Write down an expression, in terms of $p$, for the probability that he wins exactly four games. [2 marks]

(c) Hence, find the values of $p$ such that the probability that he wins exactly four games is 0.15. [3 marks]

3. [Maximum mark: 5]

Jan plays a game where she tosses two fair six-sided dice. She wins a prize if the sum of her scores is 5.

(a) Jan tosses the two dice once. Find the probability that she wins a prize. [3 marks]

(b) Jan tosses the two dice 8 times. Find the probability that she wins 3 prizes. [2 marks]
IB Review 9 – Binomial and Normal Distributions

4. A company makes containers of yogurt. The volume of yogurt in the containers is normally distributed with a mean of 260 ml and standard deviation of 6 ml.

A container which contains less than 250 ml of yogurt is underfilled.

(a) A container is chosen at random. Find the probability that it is underfilled. [2]

The company decides that the probability of a container being underfilled should be reduced to 0.02. It decreases the standard deviation to $\sigma$ and leaves the mean unchanged.

(b) Find $\sigma$. [4]

The company changes to the new standard deviation, $\sigma$, and leaves the mean unchanged. A container is chosen at random for inspection. It passes inspection if its volume of yogurt is between 250 and 271 ml.

(c) (i) Find the probability that it passes inspection.

(ii) Given that the container is not underfilled, find the probability that it passes inspection. [6]

(d) A sample of 50 containers is chosen at random. Find the probability that 48 or more of the containers pass inspection. [4]

5. A van can take either Route A or Route B for a particular journey.

If Route A is taken, the journey time may be assumed to be normally distributed with mean 46 minutes and a standard deviation 10 minutes.

If Route B is taken, the journey time may be assumed to be normally distributed with mean $\mu$ minutes and standard deviation 12 minutes.

(a) For Route A, find the probability that the journey takes more than 60 minutes. [2 marks]

(b) For Route B, the probability that the journey takes less than 60 minutes is 0.85. Find the value of $\mu$. [3 marks]
IB Review 9 – Binomial and Normal Distributions

1. (a) \((0.5)(30) = 1.5\) \(A1\)

(b) evidence of recognizing binomial probability \(M1\)

correct formula: probability = \(\binom{30}{5}(0.05)^5(0.95)^{25}\), \(X \sim B(30, 0.05)\) \(A1\)

probability = 0.0124 \(N3\)

(c) evidence of cumulative density from 0 to 4, e.g. binomial cdf to 4, \(P(X < 5)\) or \(P(X \leq 4)\)

\(\text{binomialcdf}(30, 0.05, 4)\) or \(\sum_{n=0}^{4} \text{binomialpdf}(30, 0.05, n)\) \(A1\)

probability = 0.984 \(N3\)

2. (a) evidence of recognizing binomial probability (may be seen in (b) or (c)) \(M1\)

e.g. probability = \(\frac{7}{4}(0.9)^7(0.1)^3\), \(X \sim B(7, 0.9)\), complementary probabilities

probability = 0.0230 \(A1\) \(N2\)

(b) correct expression \(A1A1\) \(N2\)

e.g. \(\frac{7}{4} p^7(1 - p)^3, 35 p^4(1 - p)^3\)

Note: Award \(A1\) for binomial coefficient \(\binom{7}{3}\), \(A1\) for \(p^7(1 - p)^3\).

(c) evidence of attempting to solve their equation \(M1\)

e.g. \(\frac{7}{4} p^7(1 - p)^3 = 0.15\), sketch

\(p = 0.356, 0.770\) \(A1A1\) \(N3\)

[7 marks]

IB Review 9 – Binomial and Normal Distributions

QUESTION 3

(a) 36 outcomes (seen anywhere, even in denominator) \(A1\)

valid approach of listing ways to get sum of 5, showing at least two pairs e.g. \((1, 4)(2, 3), (1, 4)(4, 1), (1, 4)(4, 1), (2, 3)(3, 2)\), lattice diagram

\(P(\text{prize}) = \frac{4}{36} = \frac{1}{9}\) \(A1\) \(N3\)

(b) recognizing binomial probability \(M1\)

e.g. \(B(8, \frac{1}{9})\) binomial pdf, \(\text{binomialcdf}(8, \frac{1}{9}, 8)\)

\(P(3 \text{ prizes}) = 0.0426\) \(A1\) \(N2\) [5 marks]

5. \(A \sim N(46, 10^2)\) \(B \sim N(\mu, 12^2)\)

(a) \(P(A > 60) = 0.0808\) \(A2\) \(N1\) [2 marks]

(b) correct approach \(A1\)

e.g. \(P\left(Z \leq \frac{60 - \mu}{12}\right) = 0.85\), sketch

\(60 - \mu = 1.036...\) \(A1\)
\\(\mu = 47.6\) \(A1\) \(N2\) [3 marks]
4. (a) $0.0477903$

probability = 0.0478

$A2 \quad N2$

[2 marks]

(b) $P(\text{volume} < 250) = 0.02$

$z = -2.05374$ (may be seen in equation)

$A1$

attempt to set up equation with $z$

$M1$

$\frac{\mu - 260}{\sigma} = z \cdot 260 - 2.05(\sigma) = 250$

$4.86914$

$\sigma = 4.87$ (ml)

$A1 \quad N3$

[4 marks]

(c) (i) $0.968062$

$P(250 < \text{Vol} < 271) = 0.968$

$A2 \quad N2$

(ii) recognizing conditional probability (seen anywhere, including in correct working)

$R1$

$P(A|B) \cdot P(A \cap B) = P(A|B)P(B)$

$A1$

correct value or expression for $P(\text{not underfilled})$

$0.98, 1 - 0.02, 1 - P(X < 250)$

$0.987818$

$A1$

probability = 0.988

$[6 marks]$