ACCELERATED PHYSICS—SUMMER HOMEWORK
All physics students need to be familiar with a range of mathematical techniques. The abilities described here are not prerequisites for undertaking a Diploma Programme physics course, but they do represent the skills expected of examination candidates by the end of such a course.

In order to be successful in any IB level physics class, students must be able to meet these mathematical requirements. The summer homework is designed to ensure a basic understanding of some of the most frequently used mathematical processes and procedures.

Some of this may be review, and other parts may be new. It is expected that you have a baseline understanding of the concepts covered here. If there are sections that are unclear, or difficult to understand, that is okay—we’ll clarify that at the beginning of the year. Do your best to understand each concept. Getting something incorrect isn’t a big deal, not trying it is.

This assignment should be completed over the summer and is due the first week back in school.

Arithmetic and computation
• Make calculations involving addition, subtraction, multiplication and division.
• Recognize and use expressions in decimal and standard form (scientific) notation.
• Use calculators to evaluate: reciprocals; roots; logarithms to base 10 (lg); powers; arithmetic means; degrees; radians; natural sine, cosine and tangent functions and their inverses.
• Express fractions as percentages and vice versa.

Algebra
• Change the subject of an equation by manipulation of the terms, including integer and fractional indices and square roots.
• Solve simple algebraic equations
• Substitute numerical values into algebraic equations.
• Comprehend the meanings of (and use) the symbols /, <, >, ≥, ≤, â˜‰, ≈, |x|, µ, Δx.

Geometry and trigonometry
• Recall the formulae for, and calculate areas of, right-angled and isosceles triangles, circumferences and areas of circles, volumes of rectangular blocks, cylinders and spheres, and surface areas of rectangular blocks, cylinders and spheres.
• Use Pythagoras’ theorem, similarity of triangles and recall that the angles of a triangle add up to 180° (and of a rectangle, 360°).
• Understand the relationship between degrees and radians, and translate from one to the other.
• Recall the small-angle approximations.
Significant Figures

Significant figures can be annoying. It involves how exact your measurements are, and how to calculate them. If I measured the distance from school to my house, and said I lived 8.5 km from school...that would be a good estimate. I may live 8.48 km, or 8.53 km, but I don’t have the ability to measure to that level of precision. So I use 8.5. Here’s where it gets tricky....those two significant figures play into the rest of my calculations as well.

If I said I walked to my house, then 0.02 km longer, those 0.02 km are smaller than my range of accuracy. So, ultimately, they wouldn’t count. Or if I said I walked to my house then another 0.73 km farther...I would still need to round to one decimal place, because that is the least accurate place value. Significant Figures allow us to ensure accuracy in our measurements. The following rules will help determine how to identify and use significant figures. Yes, we know it can be tedious and annoying. However, is a necessary evil of physics calculations 😊

1. All non-zero digits are significant.
ex: 112.6 m =>4 sig digs

2. All zeros between non-zero are significant.
ex: 108,005 => 6 sig digs

3. Numbers that end with zeros to the left of the decimal place need further explanation to determine # of sig figs ex: 200 => ????

4. All zeros to the right of a decimal but to the left of a non-zero digit are not significant (place holders) ex: 0.000641 => 3 sig digs

5. All zeros to the right of a decimal and following a non-zero digit are significant.
ex: 0.7000 => 4 sig digs 20.00 => 4 sig figs

6. Handling #’s when multiplying and dividing use the # of figures in the # with the fewest.
Ex: 2.0 x 2510 x 1.000 = 5.0 x 103

7. Handling #’s when adding and subtracting use largest place value present in the factors
ex: 0.20 + 10.3 + 190.600 = 201.1

Determine the number of significant digits in each number, using Rules 1-5:

22.8 ________ 1.00 _________ 5 _________ 0.00098 ________

500 _________ 500.0 __________ 1.3 _________ 1.300 __________

Multiply and divide the following numbers, keeping the proper number of significant figures, following rule #6 Ex: 22 x 1.75 =38.5 on your calculator...but 22 only has two significant figures, so there may only be two significant figures in your answer. 38.5 turns into 39.

2.4 x 0.999 = ___________ 1500 x 4.15 ________________ 2.25 x 0.0021839 = ____________

986 ÷ 5 = ______________ 986 ÷ 5.00 ______________ 300 ÷ 3.5 = ______________

Add and subtract the following numbers: (suggestion: Place them vertically and line up the decimals to determine the largest decimal place for adding)

1.4 + 225 = 226 (not 226.4...yes your calculator says that, but when using significant figures, significance doesn’t go past the ones place in 225, so the answer is not able to go past the ones place.

2.25 + 13 = ___________ 150 + 2.5 = ________________ 44 + 120 = ______________

175 – 2.1 = ___________ 450 – 25 = ________________ 18.7 – 6 = ______________
UNIT CONVERSIONS
If you were doing the following multiplication problem, you could easily cancel out numbers that existed on the top and the bottom, leaving you with 1/7 as your answer. No problem, right?

\[
\frac{6}{9} \times \frac{5}{6} \times \frac{1}{5} \times \frac{9}{7} = \frac{1}{7}
\]

In unit conversion, you do the same thing...sort of 😊 When converting from one unit to another, you multiply and divide by 1, and then cancel units out. Here’s how it works.

I want to find out how many centimeters exist in 100 km. I know there are 1000 meters in 1 kilometer.

So I multiply 100 km by \( \frac{1000 \text{m}}{1 \text{km}} \).

This allows me to cancel out the km in the numerator and the denominator, leaving meters for my unit. (100 km = 100,000 m) We want to get to cm. So we multiply by another conversion unit. There are 100 cm in 1 m, so we multiply by \( \frac{100 \text{cm}}{1 \text{m}} \).

This allows us to cancel out the meters in the numerator and the denominator, leaving us with cm. (100 km = 100,000 m = 10,000,000 cm)

It looks like this:

\[
100 \text{km} \times \frac{1000 \text{m}}{1 \text{km}} \times \frac{100 \text{cm}}{1 \text{m}} = 10,000,000 \text{cm}
\]

A few things to remember:

- 1000 m = 1 km
- 100 cm = 1 m
- 1000 mm = 1 m
- 60 sec = 1 min
- 60 min = 1 hour
- 3600 sec = 1 hour
- 1 mile = 1.609 km
- 1 meter = 3.28 feet

Change the following measurements into the new units:

- 23.4 km = __________ meters
- 15 miles = _________ km
- 72 feet = __________ m
- 3 hours = __________ seconds
- 14 seconds = ________ hr
- 65 mi/hr = ________ m/s
**SCIENTIFIC NOTATION**

Written in the form: \( M \times 10^n \)

\( 1 < M < 10 \) (M is a number between 1 and 10)

\( n = \# \) of places the decimal moved in order to make the \( M \) value. If \( n \) is negative, you’ll move the decimal to the left. If \( n \) is positive you’ll move the decimal to the right.

- \( 12,345 \): \( M = 1.2345 \) \( n = 4 \)
  - Answer: \( 1.2345 \times 10^4 \)
- \( 0.00364 \): \( M = 3.64 \) \( n = -3 \)
  - Answer: \( 3.64 \times 10^{-3} \)

If the value you are looking for falls between 1 and 10 already, you would write it as \( M \times 10^0 \)...because you would move the decimal 0 places. (and because \( 10^0 = 1 \))

The nice part about scientific notation is that it makes significant figures easier to use. If it’s written in the \( M \) value, it’s significant. \( 2.3 \times 10^7 \) has two significant figures. \( 2.30 \times 10^7 \) has three significant figures. \( 1.5 \times 10^{-4} \) has two significant figures...see? easy!

Change the following numbers into scientific notation.

- \( 2300 = \ldots \times 10^\ldots \)
- \( 0.00089 = \ldots \times 10^\ldots \)
- \( 347,000 = \ldots \times 10^\ldots \)

Change the following numbers out of scientific notation.

- \( 1.5 \times 10^7 = \ldots \)
- \( 1.22 \times 10^0 = \ldots \)
- \( 9.20 \times 10^{-4} = \ldots \)

**TRIANGLES AND TRIGONOMETRY**

We will do a lot of work with breaking measurements into separate components. A functional understanding of trigonometry is necessary. The following processes will help refresh your memory from math class.

- **Pythagorean Theorem**
  - \( A^2 + B^2 = C^2 \) for any right triangle

  Given any right triangle, with two of the sides known, you should be able to solve for the third side. Remember, the hypotenuse (\( C \)) is always the longest side

Also remember, **SOH CAH TOA**, where

- \( \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \)
- \( \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \)
- \( \tan \theta = \frac{\text{opposite}}{\text{adjacent}} \)
Please determine the missing length, using the Pythagorean theorem.

Please determine, and fill in, all the sides and angles of each triangle.
**UNCERTAINTY CALCULATIONS**

No measurement is perfect, but we can be confident in our measurement, within a given uncertainty. This next section gives practice in calculating with uncertainty. It is a technique we will be using regularly, especially in lab work.

**Uncertainty**
- Uncertainties in measurements can be stated as absolute values or percentages
- Ex 1.0±0.5 or 1.0±50%

**Absolute uncertainties are 1 sig fig and match the place value of the least significant digit of the measurement**

**Determining Uncertainties**
- A simple approximate method is sufficient to determine maximum uncertainties.
- For functions such as addition and subtraction, absolute uncertainties may be added.
- For multiplication, division and powers, percentage uncertainties may also be added.

**Adding and Subtracting**
When adding, numbers will be added, as will uncertainties. Sig fig rules apply for adding the numbers, but the uncertainty will only have one sig fig.

Ex. 1: 4.50±0.01 + 3.22±0.01

\[
4.50 + 3.22 = 7.72 \\ 0.01 + 0.01 = 0.02
\]

Answer: 7.72±0.02

• When subtracting, numbers will be subtracted, but uncertainties will be added.

Ex. 2: 5.60±0.01 - 3.20±0.02

5.60 - 3.20 = 2.40 \\ 0.01 + 0.02 = 0.03

Answer: 2.40±0.03

a) 12.5±0.5 + 6±1 = 19±2
b) 12.005±0.001 - 0.039±0.001 = 11.966±0.002
c) 75,000±1000 + 5550±50 = 81,000±1000
Multiplying and Dividing

When multiplying, percent uncertainty may be added. (Absolute uncertainties must be changed into percent uncertainties)

• Ex 3: $15.0\pm0.5 \times 8.0\pm0.5$

$15.0 \times 8.0 = 120$

Change absolute uncertainty to percent uncertainty

$0.5 \div 15.0 \times 100 = 3.333\%$

$0.5 \div 8.0 \times 100 = 6.250\%$

Add percent uncertainties

$3.333\% + 6.250\% = 9.583\%$

Turn percent uncertainties into absolute uncertainty

$120 \times 0.09583 = 11.4996 \rightarrow 10$ (Uncertainty goes to one sig fig for final answer.)

**Answer: 120±10**

• Ex4: $12.5\pm0.5 \div 62\pm1$

$12.5 \div 62 = 0.20$

• Change absolute uncertainty to percent uncertainty

$0.5 \div 12.5 \times 100 = 4.00\%$

$1 \div 62 \times 100 = 1.613\%$

Add percent uncertainty

$4.00 + 1.613 = 5.613\%$

Turn percent uncertainties into absolute uncertainty

$0.20 \times 0.05613 = 0.011$

Uncertainty goes to one sig fig for final answer.

**Answer: 0.20±0.01**

Practice

• $12\pm8\% \times 2.3\pm12\% = 28\pm20\%$

• $400\pm10 \div 0.55\pm0.01 = 730\pm30$

Complete the following calculations, with the proper uncertainty.

1. $5.0\pm0.2 + 6.0\pm0.3 = 7.95\pm0.5 \times 12\pm0.2 =$

2. $4.9\pm0.1 + 1.1\pm0.3 = 8.18\pm2 \times 0.5\pm0.01 =$

3. $1.0\pm50\% + 2.0\pm30\% = 9.25\pm5\% \times 1.5\pm12\% =$

4. $45\pm5 - 3.4\pm0.4 = 10.140\pm4 \div 12\pm1 =$

5. $1.2\pm0.4 - 0.92\pm0.02 = 11.45.5\pm5 \div 16\pm2 =$

6. $72\pm13\% - 17\pm6\% = 12.76\pm5\% \div 154\pm2\% =$

11. $45.5\pm5\% \div 16\pm2 =$

12. $76\pm5\% \div 154\pm2\% =$